

MERCATOR'S SAILING,

Applied to the True

FIGURE of the EARTH.

WITH AN

INTRODUCTION,

CONCERNING THE

Discovery and Determination of that FIGURE.

By *PATRICK MURDOCH*, M. A. *K*
Rector of *Stradishall* in *Suffolk*.



L O N D O N :

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MECHANICAL'S SAILING

Applied to the Line

FIGURE OF THE EARTH

WITH AN

INTRODUCTION

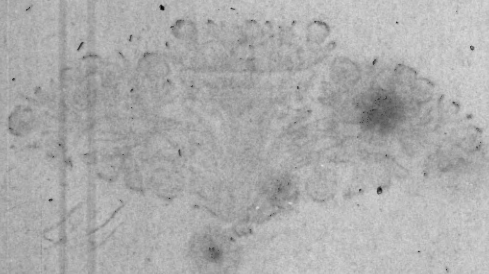
CONTAINING THE

DIFFERENCE AND DISCREPANCY



BY P. A. R. I. S. M. U. R. D. O. W. M. A.

Revised and corrected in 1844



LONDON

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TO THE
RIGHT HONOURABLE
THE
Lord GEORGE GRAHAM.

MY LORD,

THE Subject of this Essay being nearly related to the Profession in which you chuse to serve your Country, I have taken the Liberty to address it to your Lordship: And am proud of this Opportunity of publickly declaring with what unfeigned Respect and Gratitude I am,

MY LORD,

Your Lordship's

Most Obliged, and most

Obedient humble Servant,

PATRICK MURDOCH.

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THOMAS MURDOCH

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INTRODUCTION,

Concerning the Magnitude and Figure of the Earth.

I.

WHOEVER proposes any Correction of an established Rule, ought, in Reason, to give some Account of the Grounds on which he proceeds. And although, with Regard to the following Essay, I might the more easily be excused this Trouble, that the Principle which it supposes, is now generally known and admitted ; yet some, who are less conversant in the Authors who treat of these Matters, may not be displeased to see, in one distinct View, what has been formerly believed, or is known to be, the true Figure of the Earth.

THE first Men, whilst they lived together on their native Soil, must have imagined the Surface of the Earth to be one immense Plain, diversified by Mountains, Valleys, Beds of Rivers, and the like. Afterwards, when they began to disperse themselves, and make Excursions into distant Countries, the different Elevation of the Heavenly Bodies above their new Horizon, the different Length of the Days and Nights, and different Degrees of Heat and Cold, could not fail to excite their Attention and Curiosity : Yet these Appearances might not at first suggest to them any real Change of their Horizon, or that the Surface of the Earth was *convex*. They had ob-

observed, no doubt, that the top of a Mountain, or of a Tower is elevated as one approaches it; and the contrary. This might, to them, account for the Elevation or Depression of the heavenly Bodies; as a supposed Difference of Distance from the Sun might for the Degrees of Heat and Cold. Nor can it be gathered, but from Conjecture, when, or by whom the Discovery of the Earth's Convexity was made.

THAT this Opinion is very ancient, cannot be doubted. *Thales* the *Milefian*, about 600 Years before the Birth of *Christ*, could foretel Eclipses *; which shews that the Earth's spherical Figure was known, not only to him, but to the Astronomers of preceeding Ages, from whose Observations a Theory of the celestial Motions had been composed. His Disciple *Anaximander* even attempted to measure the Earth's Circumference.

THIS Knowledge *Thales* might derive either from the *Phenicians*, or from the *Egyptians*; both which Nations he had visited. The former, whose Business was Trade and Navigation, could not fail to deduce it from this single Observation, though they had made no other; That in sailing from any high Coast, the lower parts of the Land are gradually hid by the Surface of the Water, till at length the whole disappears; That, on the contrary, in making towards Land, the tops of Mountains are first discovered, and the rest gradually as the Ship advances.

OR may we not, with some reason, imagine that the *Egyptians* had very early made the like Observations at home? If they did not go to Sea, the annual Return of the *Nile* brought the Sea to them. The Country was overflowed to a considerable depth, their Barks rode in the Streets of the Towns, and the Towns themselves rising from the watery Surface gave a Prospect like that of the *Egean* Sea with its Islands †. Here then there wanted neither a Horizon, nor Objects proper for Observation; nor an acute speculative People at leisure to profit of them.

* Herodot. Lib. I.

† Id. Lib. II.

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HOWEVER this be, as soon as Geometry was cultivated, and it was found that by giving the Earth a spherical Figure, the Appearance now mentioned, and all others in Geography were conveniently solved, it was reasonably concluded that its Figure could be no other than Spherical.

THIS naturally introduced the Enquiry concerning its Dimension and Magnitude: An Enquiry which was very early begun, and is not quite finished at present. For this purpose various Methods were thought of; all equally simple and accurate in the *Theory*, but difficult in the *Execution*, and sometimes impracticable.

OF the latter sort may be reckoned those Methods, which depend on Terrestrial Observations only. As when, from the top of a high Mountain, the Angle which a Line drawn to the extreme Verge of the Horizon makes with the perpendicular, is measured; and from this Angle with the height of the Mountain the Semidiameter of the Earth is computed. Such Methods, I say, are impracticable, or at least insignificant. Because it is difficult to find exactly the perpendicular Height or the Distances of Mountains, and no less so to measure the Angle that is wanted, even with the best Instrument; not to mention that the Altitude of the highest Mountain bears so small a Proportion to the Earth's Semidiameter, that any Error in measuring the one, is greatly multiplied in determining the other.

IN the Methods which have been used with better Success, the Distance of two Places on the same Meridian is supposed known, and likewise the different meridian Altitudes of a fixt Star at both places; and from a Comparison of the given Distance with the Difference of Altitude, the Proportion of that Distance to the Circumference of the Earth is found. The Sun may be used instead of a fixt Star, if his meridian Altitude is taken at both Places on the same Day.—But this will be best explain'd by the help of *Fig. I.*

IN which CABN represents a meridian Circle of the Earth, that is a Circle wherein a Plane drawn through the Centre and the Poles would cut the Globe. A and B are two Places in the Circumference

a

of

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of this Circle, whose Distance AB is known; AD , GBD , touching the Circle in A and B , are Lines in which the Plane of the Meridian cuts the Horizons of the two Places; and SA , SB , are two Lines, or visual Rays from a Star to the Places A and B . It is likewise taken for granted or proven,

1. THAT the Arc AB hath the same Proportion to the whole Circumference $ABNA$, as the Angle ACB to four right Angles. 6 *El.* 33.

2. That ZA , a Plumb-Line at A , hangs perpendicular to the Horizon AD , as does the Plumb-Line zB to the Horizon GBD . Whence ZA , zB produced, will meet in the Centre C . 3 *El.* 19.

3. THAT the Distance of a Star is incomparably greater than the Diameter of the Earth; and that therefore the Lines SA , SB may be taken for parallel.

Now if two Observations of the Star's meridian Altitude are made at A and B , these Altitudes will be measured by the Angles SAD , SBG ; and if BE , BF are drawn parallel to AZ , AD , seeing likewise SA is parallel to SB , the Angle SAD will be equal to SBF ; whence the Angle FBG will be the Difference of the Altitudes observed. But because zBG , EBF are Right-Angles, if zBF common to both is taken away, there will remain FBG equal to zBE ; and BE being parallel to CAZ , the Angle zBE is equal to BCA , that is, BCA is equal to the Difference of the Altitudes observed. Say therefore, As that Difference is to four right Angles, so is the Arc AB , in any known Measure, to the whole Circumference in the same Measure.

It had been the same thing, if instead of the Altitudes above the Horizon the Distances from the Zenith SAZ , SBz had been observed; for their Difference would have been zBE equal to BCA , as before.

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To this Method, or others reducible to it, we owe all the Measures of the Earth's Circumference, that have been taken by Authors of any Note, from *Eratosthenes* down to the present Times. The most remarkable of which I have transcribed from *Varenius* and *de Maupertuis*, and reduced as follows.

THE Circumference of the Earth contains according to

	Stadia or Furlongs.	Roman Miles of 8 Stadia.
<i>Eratosthenes</i>	250000	31250
<i>Hipparchus</i>	275000	34375
<i>Posidonius</i>	240000	30000
<i>Strabo and Ptolemy</i>	180000	22500
<i>The Arabians</i>		20340 Miles.

According to

	Rhinland Perches of 12 Feet.	French Toises or Fathoms.	English Sta- tute Miles of 5280 Feet.
<i>Norwood</i>	10701219	20679000	25036 $\frac{1}{4}$
<i>Picart</i>	10630116	20541600	24369 $\frac{1}{8}$
<i>Muffchenbroeck</i>	10625107	20531920	24858

Any of which Numbers multiply'd by the Fraction $\frac{8}{355}$ will give the Diameter in the same Measure; or divided by 360 will give the Length of a Degree.

Note. THE Paris Foot is to the English nearly as 38,355 to 36; the Rhinland Foot to the English as 105 to 102; and the ancient Roman Foot to that of Paris, as 11 to 12. *

* *Histoire de l'Academie de Scien.* Ann. . . .

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IN this Table it may be observed, 1°. That the Numbers of the Ancients differ widely from one another; which has made it suspected that their *Stadia*, or Furlongs, were not of the same length, as indeed it would be difficult to prove that they were. *Vitruvius* and *Pliny*, with most Authors, give 8 *Stadia* to a Roman Mile of 1000 Paces, or 5000 Feet. But Mr. *Cassini*, from a Passage in *Strabo*, finds, that at least 9 *Stadia* were contained in the same Mile in the South of *France* *. And the Dimensions given by Authors of the greatest of the *Egyptian* Pyramids only encrease the Uncertainty.

BUT allowing that the ancient Geographers by a *Stadium* do every where mean one determinate known Measure, as it was most natural they should, their disagreement, in the present Case, is sufficiently accounted for from the bad Instruments and little Care they used in Operations of this kind. How ill provided they were of Instruments we see from the *Arenarius* of *Archmedes*, where we find the apparent Diameter of the Sun taken by means of a Cylinder placed at such a Distance from the Eye as just to cover his Body. And *Eratosthenes*, in his famous Observation, had only a hollow Hemisphere with a Gnomon erected in it, whereby to measure the Sun's Distance from the Zenith of *Alexandria* †. Besides, they took the Distances of Places not from an actual Survey, but from common Estimation; and were not very solicitous whether they lay precisely in the same Meridian.

THE Moderns have performed this Work with greater Care, and, we see, to better purpose. Upon the revival of Arts, large well-graduated Instruments were made, the Distances of Places were exactly measured, and proper Allowances made for all Deviations, both from the Horizontal Line, and from the Meridian. At length Telescopic Sights were applied to Instruments, by which means Angles, whether above or in the Horizon, could be taken to a much greater precision; and the Instrument itself could be better examined and rectified. Still it was troublesome to make Surveys

* *Histoire de l'Academie de Scien.* Ann. . . .

† *Vid.* Fragment. post Aratum. Edit. Oxon.

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of a proper Length, such as Mr. *Norwood's*, who measured the Distance from *London* to *York* with a Chain. To remedy this, it was found sufficient, or even better, to measure with the greatest Accuracy some smaller Distance as a *Base*, from which by means of a Series of Triangles connected with it, an Arc of the Meridian of a competent Length might be deduced.

FOR it is not difficult to conceive, that in a Figure consisting of any Number of Triangles, all whose Angles are known, and whereof every two that are contiguous have one Side common, if any one Side is given in length, the Dimensions of the whole Figure may be found by *Trigonometry*; and its Position with respect to the Meridian from an *Astronomical* Observation. Whence, if a Meridian-Line is supposed to pass through any Angle of the Figure, and perpendiculars to be drawn to it from the other Angles, the Intercepted Segments will be known: By which means an Arc of the Meridian of any Length may be measured. Only these things are to be noted; That 'tis convenient the Triangles be as few as possible; that they be continued nearly in the Direction of the Meridian, and that the Stations or angular Points, whether Spires of Churches, &c. or Signals erected on purpose, be so situated, that none of the Angles be very small. But all this will be best understood from Mr. *de Maupertuis* Figure of the Earth determined; where the whole Process is particularly described, and illustrated with proper Schemes.

II.

HITHERTO the Figure of the Earth has been considered as perfectly *Spherical*; to which indeed it so nearly approaches, that the Difference was not to be discovered in the Method already explained, if the Arcs measured were nearly in the same Latitude, especially when no such Difference was supposed or look'd after. For although *Norwood*, *Picard*, or ever so many good Artists in *France* and *England*, had, by good luck, each assigned the just Length of the Degree which he measured, the Differences of these Lengths would have been no greater than might be reasonably attributed.

attributed to the Errors that are unavoidable in such Operations. And accordingly, however various the Length of a Degree, as rated by different Authors, had been, there never arose the least Suspicion of the Earth's being other than a perfect Sphere, till Mr. *Richer's* celebrated Observation in 1672; "That the same Pendulum vibrates slower near the Equator than it does in *France*." Then indeed, the true Figure of the Earth could not be long unknown; this *Phenomenon* happening in the time of *Huygens* and *Newton*, the two Persons best qualified to reason upon it; and, by a fate peculiar to it self, scarce sooner appearing than it received a just Solution.

BUT in order to give some intelligible Notion of this matter, and to shew the Relation there is between the Motion of a Pendulum and the Figure of the Earth, it is necessary first to explain what is meant by *Centrifugal Force*; a Term of principal Use on this Occasion.

IT is a known Law of Motion, that a Body impelled by another moves on equably in the Direction of the Impulse; and also if the Impulses be two, three, or any finite Number, the Body upon which they are exerted, will still move in a strait Line, with a Velocity and Direction resulting from the Quantities and Directions of all the several Impulses.

WHENCE it follows, that when a Body moves in a Circle, or other Curve-Line, it is retained in that Curve by some force acting upon it *continually*. Such is the Force which the Hand exerts upon a Stone whirled round in a Sling; and such is that conceived to be, by which the Heavenly Bodies are retained in their Orbits.

Action being ever equal and contrary to *Reaction*, if we consider the *Tension* of the Sling as produced by a Force acting towards the Hand as a Centre, then is that Force called *Centripetal*: But if we consider it as a Force in the Stone, acting in a contrary Direction, it is called *Centrifugal*. And in all Bodies moved round a Centre or Axis, such a centrifugal Force really is, although its Effect is not always discernible.

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IN solid Bodies revolving round an Axis, every Particle tends to recede from the Centre, but the Effect is destroyed by the stronger Cohesion of the Parts; 'tis otherwise in those that are fluid or very soft; as we may see in the circular Motion of a Bucket of Water suspended by a String, in the Motion of a Grindstone, and many other Instances that are familiar to every one.

ONE Law of centrifugal Force is, that in the same or equal Particles of Matter, supposing likewise the times of Revolution equal, its quantity will be proportional to the Semidiameter of the Circle described. Whence we may easily gather, that *in the Circumvolution of fluid Bodies, the total Effect, or the Sum of the centrifugal Forces is proportional to the Square of the Radius of Circumvolution.*

FOR if in the Right Line CB (*Fig. II.*) there be taken, CA, CB equal to the Lengths of two small Tubes containing a homogeneous Fluid, and which are supposed to revolve, in the same time, round a Centre C: And if from A and B there be drawn at right Angles the Lines AE, BD, proportional to CA, CB; then will CED be a right Line; and any Line *pt* parallel to AE, and meeting the Sides of the Triangle in *p* and *t*, will represent the centrifugal Force at the Distance *pC*. Now if this centrifugal Force is multiplied into a given Particle of the Fluid *pq*, the Rectangle *qt* will be proportional to the *vis motrix*, or *momentum* of that particle. And the Sum of all these Momenta will be as the Sum of all the little Rectangles *qt*, that is, as the Area of the Triangle CAE. In like manner, the Sum of all the *Momenta*, or centrifugal Forces in the longer Tube will be as the Area CBD; which Areas are in the duplicate Ratio of the Sides CA, CB. 6 *El.* 19.

To confirm this, the following Experiment may be made. Let a Brass Tube of the Shape that is represented in *Fig. III.* be filled with Mercury, or with some tinged Liquor, to the height of the Line BD, and then fixed to a Machine which turns horizontally with an equable Velocity, but so as the Axis of the Leg DC, may coincide with the Axis of Motion. Then will the Fluid in the one Leg sink from

from D to E, and rise in the other from B to G; and FG, the Difference of the Heights of the Columns AG, CE, will be as the Sum of the centrifugal Forces in the Radius of Revolution CA; as is plain from *Hydrostatics*. This Difference FG may be measured on a fixt Gage contrived for that purpose, if a piece of Glafs is cemented along an Opening made in the Leg AG, through which the Fluid may be seen.

THE Experiment may be varied several ways; as in *Fig. IV.* where the Tube is supposed to revolve about an Axis dividing AC unequally, in *c*. Then ought the Difference of the Heights AG, CE, to be proportional to the Difference of the Squares of *Ac*, *cC*; making some allowance for the Tenacity and Adhesion of the Fluid. And there must now be adjusted another Tube *cS*, directly in the Axis of Motion, whereby the Fluid that rises in either Leg may be supplied.

To apply this to the Subject in question; Let us, with Sir *Isaac Newton*, imagine, that in *Fig. V.* representing the Globe of the Earth, whose Axis is *Pp*, and a Diameter of the Equator *Qq*, there passes a Canal or Tube *PCQA*, filled with water from *P* to *Q*. If the Earth is a Sphere of homogeneous matter, equally gravitating to the Centre at equal Distances, and is likewise at rest, then the Fluids in the Tubes *PC*, *QC* will ballance each other. But if the Sphere is made to revolve equably upon its Axis *Pp*, the centrifugal Forces of the Fluid in the Leg *CQ*, will take off some part of its Weight: Nor can the Equilibrium be restored, till so much of the Fluid in *PC* hath passed over into *AC*, as to make the whole Weight in *AC*, equal to the Sum of the Weight in *PC*, and of the centrifugal Forces. Hence it appears,

1. That the solid Earth at present is not Spherical; else the diurnal Revolution would throw the *Ocean* upon the Parts about the Equator, causing a general Inundation for a great Tract on both sides of it; leaving those towards the Poles, if not quite dry, at least with Coasts of an immense height above the Surface of the Ocean.

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2. THAT if the Earth was originally fluid and spherical, the communicating to it its motion of diurnal Rotation, must necessarily raise its Parts higher with respect to the Axis, and depress the Parts towards the Poles, bringing them nearer to the Centre, till it settled into the Figure called an *Oblate Spheroid*, generated by the Revolution of an *Ellipse* round its lesser Axis. Which having been demonstrated by others, I do not at present insist on.

NEITHER will this Figure be sensibly altered from that of a geometrical Spheroid, although, as Sir *Isaac Newton* supposes, the Earth should be somewhat denser towards the Centre. If we consider this redundant Matter, which makes the excess of Density, as a distinct Mass, similar in Figure to the Earth, and acting upon the more homogeneous Mass towards the Surface, in a reciprocal duplicate Ratio of the Distances, it is plain the more distant Matter towards the Equator will be comparatively lighter, than if there was no such Mass, and consequently will rise higher. But this rising at the Equator, on account of the Density at the Centre, is confined to narrow Limits; and there must be a regular Subsiding, towards the Poles, correspondent to it, which will still keep the Figure of the Earth nearly to that of a perfect Spheroid.

BUT to return; The Effect of centrifugal Force, with respect to Gravitation, is two-fold: 1°. By it the Weight of Bodies is diminished *immediately*; most at the Equator, and in any given Latitude, in a duplicate Ratio of its *Cosine*; or more rigorously as the Rectangle under the *Ordinate* and *Cosine* pertaining to that Latitude. And 2°, *mediately*, as it is the Cause of the Earth's Spheroidal Figure.

Now 'tis evident, that the Force by which a Pendulum vibrates, is no other, than that by which Bodies are attracted downwards. And that therefore if the Power of Gravitation was increased, so as a Body, for instance, which a Man could formerly raise from the Ground, should now prove too heavy for him, though he makes an equal Effort; in this case a Pendulum would move faster. And the contrary, if the Power of Gravitation was diminished. It is
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likewise

likewise true, that the time of the Vibration of a Pendulum is longer or shorter, when the Pendulum it self is so.

WHEN therefore Mr. *Richer* being in the Island of *Cayenne*, within five Degrees of the Equator, found that his Pendulum, which he had carried from *France*, went so much slower, that, to make it keep true Time, he must have shortned it by about an eighth part of an Inch, it was a certain Indication, that in those Parts the Force of *Gravitation* is less than it is in *France*. And wherever the Experiment has been repeated with due Care, the Conclusion has been the same, “*That the Power of Gravitation decreases in going from the Poles towards the Equator.*”

IT is true, the Warmth of the Climate contributed to lengthen Mr. *Richer*’s Pendulum, but not so much as $\frac{1}{8}$ of an Inch in a Rod of little more than three Foot; as is gathered from Experiments that have been made upon the Extension of Metals by Heat. Besides, the *French Academicians*, in their late curious Experiments on this Subject, contrived to keep the Air in the same Degree of Heat at *Pello*, in Lat. $66^{\circ}. 48'$. as afterwards at *Paris*; which was measured in both places by the same Thermometer.

IF therefore, from the different Lengths of a Pendulum in different Latitudes, the *Diminutions of Gravity* are found; and from the total *Diminutions* those Parts are subtracted, which result immediately from *centrifugal Force*, what remains will be the *Diminutions* that are owing to the *spheroidal* Figure of the Earth, supposing it every where of the same *Density*: Which reduces the Question to this; “What Species of a Spheroid will, from the General Law of Attraction, produce a given Diminution of Gravity at a given Point of its Surface?”

IF, for Instance, the Length of a Second-Pendulum, at the Pole, is 441.38 *Lines*, or twelfth parts of a *Paris* Inch, and at the Equator 439.468, the Difference being 1.912 *Lines* *; the Diminution

* See Sir *Isaac Newton*’s Table.

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of Gravity will be expressed by the Fraction $\frac{1 \cdot 2 \cdot 3 \cdot 4}{4 \cdot 4 \cdot 3 \cdot 3 \cdot 0}$, from which subtracting $\frac{1}{2 \cdot 8 \cdot 9}$, the *immediate* Effect of *centrifugal Force*, the remainder is $\frac{1}{11 \cdot 4 \cdot 8}$. But Sir Isaac Newton (*Prop. 19. Lib. III.*) computes, That if the Semiaxis of a Spheroid which is at rest, is to the Semidiameter of its Equator as 100 to 101, the Diminution of Gravity at the Equator will be $\frac{1}{3 \cdot 0 \cdot 1}$. Say therefore, As the Diminution $\frac{1}{3 \cdot 0 \cdot 1}$ is to the Difference $\frac{1}{1 \cdot 0 \cdot 0}$; so is $\frac{1}{11 \cdot 4 \cdot 8}$ to $\frac{1}{2 \cdot 8 \cdot 9}$: That is, the Semidiameter of the Equator is to half the Axis as 230 to 229.

IN the same Proposition it is shewn, That the Fraction $\frac{1}{2 \cdot 8 \cdot 9}$ expresses likewise that part of the Weight of the Column CA (in *Fig. V.*) which is sustained by the *centrifugal Forces*: And farther, that in the Spheroid of the last Paragraph, whose least and greatest Semidiameters are as 100 to 101, the Excess of Weight of the Fluid in the Column CA, above the Weight of CP will be $\frac{4}{3 \cdot 0 \cdot 3}$ of CA. If then the Excess of Weight $\frac{4}{3 \cdot 0 \cdot 3}$ give the Difference of Semidiameters $\frac{1}{1 \cdot 0 \cdot 0}$; what will the Excess of Weight $\frac{1}{2 \cdot 8 \cdot 9}$ give? The Answer is, $\frac{1}{2 \cdot 8 \cdot 9}$, as before. That is, in a Spheroid, whose Semidiameter of the Equator is to its Semiaxis as 230 to 229, the Excess of Weight in the Column CA, will be equal to that Weight which the centrifugal Forces can sustain; and consequently the Fluids in the Canals CA, CP will rest in *Equilibrio*.

UPON this Proportion of 230 to 229, Sir Isaac Newton calculates his *Table* of the Lengths of a Pendulum in different Latitudes. But the Differences of the *computed* Lengths coming out lesser than what they had been found by Experience, he introduces the *Mass of denser Matter*, which was already mentioned: And increasing the Difference of the Semidiameters in the same Ratio as the *observed* Difference of Length of the Pendulum exceeds the *computed*, raises the Earth from $17\frac{1}{2}$ to $31\frac{7}{8}$ Miles higher at the Equator.

OR, which will nearly serve to the same purpose, we may use the following Rule; " Instead of $\frac{1}{2 \cdot 8 \cdot 9}$, take a Fraction (*f*) which " in either of the above *Analogies* shall give the same Answer." Which is equivalent to this Supposition, " That the Effects of the

" *denſer Maſs*, viz. the greater Diminution of accelerating Force,
 " and the Elevation at the Equator correſponding to it, are *nearly*
 " the ſame, as if they had been produced by a *ſwifter diurnal Ro-*
 " *tation*."

THUS if, according to Monfr. *de Maupertuis's Table* *, the Length of a ſecond-pendulum is at the Pole $441\frac{1}{2}$ Lines, and at the Equator 439.33, the Difference being 2.17; the whole Diminution of Gravity will be $\frac{2.17}{441.5}$, from which if we take $\frac{1}{184\frac{1}{2}}$, and work by the former *Analogie*, the Ratio ſought will be $\frac{1.38}{1.37}$, which is too great, as $\frac{2.30}{2.29}$ is too ſmall. But if, for $\frac{1}{184\frac{1}{2}}$, we uſe the Fraction $\frac{1}{184.7}$, either *Analogie* will give the Ratio of the greateſt to the leaſt Semidiameter, as 203 to 202.

Now this *Table* of Monfr. *de Maupertuis* is it ſelf calculated upon a *Hypotheſis* which had been already given up; namely the uniform Denſity of the Earth; and differs in effect from Sir *Iſaac's* in this only, that the *greater obſerved* Difference of Length, between *Paris* and *Pello*, upon which the Calculus proceeds, does of conſequence affect the whole *Table*. We may therefore ſuppoſe the Fraction $\frac{2.17}{441.5}$ is too ſmall. And if inſtead of it we take $\frac{3.00}{441.5}$, we ſhall not probably be much wide of the Truth. For, by the beſt Obſervations in Sir *Iſaac Newton's* Time, the Difference from the Equator to *Paris* was about 2 Lines; from *Paris* to *Pello*, by M. *de Maupertuis's* own accurate Experiments, it is $\frac{6}{10}$ of a Line, and allowing an increaſe of $\frac{4}{10}$ more from *Pello* to the Pole, the Sum is three Lines. In which Caſe, the Fraction (*f*) will now be increaſed to $\frac{1}{184\frac{1}{2}}$, and the Ratio of the Semidiameters to $\frac{1.47}{1.46}$.

BUT here it muſt be obſerved, that although this *Rule*, as founded on no unreaſonable Suppoſition, may ſerve till a better is diſcovered, yet is it neither demonſtrable, nor perhaps ſufficiently exact. From the Experiments with Pendulums, we may juſtly infer an Elevation of the Equator *in general*, but not its precise Quantity; this laſt depending

* *Figure of the Earth determined.*

pending partly on the internal Constitution of the Earth, which is unknown. And admitting that the Figure of the Earth is to be determined from a Ballance of the centrifugal and attractive Powers; as also, that the Diminution of *accelerating Force* at the Equator (above what is accounted for from the *centrifugal*, and from the Attraction of a homogeneous Spheroid) is owing to the *Attraction* of a denser interior *Mass*; yet it will not *necessarily* follow, that the Equator is thereby raised higher than if the Earth were homogeneous. Its Elevation may indeed be either *lesser*, *equal* or *greater*, according to the *Conditions* of the *denser Mass*. What probable Arguments there are, that this last is the Case in Nature, shall be afterwards considered.

AT present, let us suppose a *fluid homogeneous Earth* to revolve, its greatest and least Diameters being, in this State, as 230 to 229; and that afterwards a part of the *Fluid* is converted into a *concentric solid Sphere*, with an attractive Force, such as that of the *denser Mass* is supposed to be. In this case, it is plain the *Diameter* of the Equator will be contracted; for in a Column of the Fluid which lies in the Plane of the Equator, the *new Attraction* is, to a Distance from the Centre equal to the Semiaxis, the same as in the *Polar Canal*; and consequently the Attraction of the remaining part of the Equator-Column, will increase the Ratio of its whole Weight to that of the Column in the Axis; while in the mean time the Diminution of accelerating Force at the Equator will be greater than before.

IN place of the concentric Sphere, let us substitute a solid Spheroid similar to the Earth; the Canals communicating with each other along its Surface. And if we put the Ratio of the Semiaxis to the Semidiameter of the Equator to be that of *Unity* to x , the Heights of the two Columns of the ambient *Fluid* being in the same Ratio, the Weights (produced by this *new Attraction*) of two small Portions of the Fluid, proportional to the whole Columns, and similarly situated in them, will be directly as these Portions, and inversely as the Squares of their Distances from the Centre. And *these* whole Weights being in a like Ratio; that of the Column in
the

the Axis will be to the other as 1 to z , and z^2 to 1, that is, as z to 1; consequently it will preponderate, and raise the Equator higher than in the Ratio of 230 to 229.

AND if the little Spheroid is taken of a certain intermediate Species between a Sphere and that of the homogeneous Earth, its Attraction will not alter the Ratio of the Earth's Diameters.

THE Truth of these Conclusions may be examined as follows. Let a right Line RG (*Fig. V.*) touching a Meridian at the Equator in the Point E, represent the Length of a Pendulum, that is, the actual accelerating Force of Gravitation at the Equator; and, in it, ER that part which belongs to the redundant Matter. Draw to the Centre the Line GC, and from the Point R describe the Curve RFZ, having its Ordinates RE, FV in the inverse Ratio of the Squares of their Distances from the Centre, CE, CV. So shall the Area ERFV, intercepted by the Ordinate ER at the Surface of the Earth, and any other FV parallel to it, (*i. e.* the Area $RE \times EV \times \frac{CE}{CV}$) be proportional to that part of the Weight of the Column EV, which belongs to the Attraction of the *redundant* Matter: And the Trapezium EVTG, cut off by the Ordinate FV produced, shall represent that part of the Weight of the same Column, which results from the Attraction of the uniformly dense Earth. Imagine the like Construction at the Pole, marked at the analogous Points with the smaller Letters r, e, g , &c. and make the Area RGTF equal to $rgtf$, so shall you arrive at an Equation, expressing the Ratio of the greatest and least Semidiameters, answering to the *Data* and *Suppositions* upon which you proceed.

THUS, if CV is to Cv as z to Unity, that is, if the redundant Matter has a Form similar to the Earth, the Equation produced by the Comparison of the Areas RGTF, $rgtf$, will be

$aqb \times z^3 - a + b \times q + rb \times z^2 + 1 - a \times q + r \times c + b \times z + q = 0$. In which the Semiaxis of the Earth, as also the accelerating Force at the

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the Pole, being *Unity*, z is the Semidiameter of the Equator; and (e being the Difference of the Semiaxes of the Earth, and of the redundant *Mass*) $q = e - \frac{1}{2}e^2$, $a = RG$ the accelerating Force at the Equator, $c = 1 - a + \frac{1}{2}ag$, $b = \frac{1}{3} \frac{a^2}{g}$, $r = \frac{e}{1-e} - q$.

For which, if less Accuracy is required, we may use the Quadratic Equation,

$$aqb \times z^2 - 2a + b \times q + rb \times z + 2q + r \times c + l = 0.$$

In like manner, if we suppose the redundant *Mass* to be a Sphere, whose Radius is to the Semidiameter of the Equator as t to 1, putting $a \times 1 - t^2 = b$, we shall obtain the Equation.

$$c - 1 \times t^2 + bk \times z^3 + c - 1 \times t^2 + 2b - k \times z^2 + t^2 - 3 \times c + b + 1 - k \times z + 1 = 0$$

OR, for three or four decimal places, the Quadratic

$$2 - c \times t^2 - bk \times z^2 + 2 \times k - b - ct^2 \times z + 3 - t^2 \times c + b - 2 = 0.$$

AND in all Cases, the Attraction of the redundant *Mass* will, at the Pole of the Earth, be to the whole Attraction, as $\frac{c - b \times z - 1}{\frac{z^2 - 1}{z^2} - b \times z - 1}$ to *Unity*; that is, to the Attraction of the uniformly dense Earth as $c - b \times z - 1$ to $\frac{z^2 - 1}{z^2} - c$.

WHENCE likewise the Density of this *Mass*, will, in any given Case, be known; for it will be to the Density of the homogeneous Earth, nearly as the attractive Forces at the Pole directly, and the Cubes of the Semidiameters *inversely*. See *Corol. 1. Prop. 74. Lib. I. Princip.*

FROM these Equations it appears, that if the Fraction $\frac{25}{4415}$ expresses the whole Diminution of Gravity at the Equator, that is, supposing a Second Pendulum is shorter than at the Pole by about $2\frac{1}{2}$ Lines, and the denser *Mass* to be a Sphere, whose Radius is $\frac{2}{4}$ of

of the Semidiameter of the Equator, then will the Difference of the Earth's greatest and least Semidiameters be only $\frac{1}{400}$ of the latter. If the Radius of the denser Mass is $\frac{1}{10}$, the said Difference will be very near $\frac{1}{343}$. But although the denser Mass were reduced to a Point, the Semidiameter of the Equator would not exceed the Semiaxis by above $\frac{1}{333}$.

ON the contrary, the Fraction $\frac{45}{4413}$ remaining, if the denser Mass is similar to the Earth, their like Diameters being as 1 to 4, the Elevation of the Equator will be between $\frac{1}{129}$ and $\frac{1}{130}$ of the Semiaxis. And if the Semiaxis of the denser Mass is $\frac{1}{10}$, it will rise to between $\frac{1}{108}$ and $\frac{1}{109}$. Let the interior Spheroid become incomparably small, that is, let e be greater than any Fraction whatever; then, in the first Equation, r is incomparably great, which reduces it to $z = \frac{c+b}{b} = 1 + \frac{1}{99}$. Or if e is less than any Fraction whatever, then $q = e$, and, r vanishing, if we divide by q , the Equation is $abz^3 - a + b \times z^2 + 1 - a \times z + 1 = 0$; which will give z nearly equal to $1 + \frac{1}{173}$. In like manner, if the total Diminution is put equal to $\frac{30}{4413}$, the Limits of z will be $1 + \frac{1}{60}$, and $1 + \frac{1}{287}$. *

FROM which Examples we see, that a spherical Mass must contract the Semidiameter of the Equator; and in general, what the Effect of a Spheroid of any Species would be, viz. that *cæteris paribus*, the more *oblate* it is, or the lesser its Magnitude, the higher it will raise the Column at the Equator; and the contrary. And that consequently, as was above asserted, unless the Conditions of such a Mass are given, the Species of the Earth's Figure cannot, by any Rule yet known, be *certainly* inferred from the Experiments with Pendulums.

I would not here be misunderstood, as insinuating that such Experiments are of no use in this Subject. On the contrary, it is by them

* In all this, the Quantity of centrifugal Force is put $= \frac{1}{289}$, and $b = \frac{500}{301}$. But after these Fractions have served for an Approximation, they may, if the Accuracy of the Experiments requires it, be calculated a-new, and the Operations repeated.

them chiefly that it can be clear'd of the Difficulties with which it is now embarrassed. What I mean is to shew, that we ought not to be over-hasty in concluding from them, and to point out what seems to be yet wanting before we can proceed in this enquiry. "That, besides a sufficient Number of good Observations in all Latitudes we can come at, the Laws of Attraction be further studied; what they are at any Point of the Surface, not only of a homogeneous Earth, but on the suppositions, either that it includes a given Mass of denser Matter, or is gradually more dense from the Surface to the Centre." By comparing these Experiments with the *Theory*, and with each other, we may hope to see the Figure of the Earth better determin'd, and even its internal Constitution in part discover'd.

ESPECIALLY if we call in the Assistance of Observations drawn from some other Source; of which we are not altogether destitute. For as we can from the Conditions of the *Mass*, and the Diminution of Gravity, deduce the Elevation of the Equator, in the Manner already described; *vice versâ*, if, either from the Method of the *French Academicians*, which shall be afterwards explained, or by the Observation of *Lunar Eclipses*, this Elevation can be found to any tolerable Exactness, the Magnitude of a similar Spheroidal *Mass* is had by making e the unknown Quantity in the Equation $aqbz^3 -$
 $\&c.$ which will depress it to the Quadratic $e^2 - 3e + \frac{s}{\frac{1}{2} \times s - t} = 0$, in which s and t are the Co-efficients of q and r in the Equation, before it was transformed. Thus, if by whatever means z is found $= 1.01$, and $a = 1 - \frac{3}{4} \frac{0}{4 \frac{1}{2}}$, $1 - e$, or the Semiaxis of the concentric similar *Spheroid*, will be a little more than .297. That it is either similar, or not much different, is probable; and if the Difference is considerable, it will discover itself in the Motion of Pendulums in several distant Latitudes.

ALTHOUGH, from what is above demonstrated, it appears that the greater Diminution of accelerating Force at the Equator, is not altogether inconsistent with a Diminution of its Semidiameter, *viz.*

if we suppose the denser Matter to be a Figure either perfectly *Spherical*, or little differing from it; yet the same reasoning furnishes a probable *Physical* Argument, that this is not really the Case in Nature. For in one of these Examples, where the redundant Matter was a Sphere with the Radius $\frac{1}{4}$ of the Semidiameter of the Equator, if we compute its accelerating Force at the Pole, we shall find it about $\frac{3}{8}$ of the whole; and consequently the whole Density of the concentric Sphere would be to that of the ambient Matter as 42 to 1. Proportions which will not, I presume, be thought very *natural*; whereas, if the redundant Mass is a Spheroid similar to the Earth, their like Diameters being as 1 and 4, its accelerating Force at the Pole will be only $\frac{4}{8}$, and the whole Density of the Spheroid to that of the ambient Matter, in little more than the Ratio of 1307 to 1000. And the like might be shewn upon other Suppositions of Magnitude; whence we may reasonably infer, that the Figure approaches much nearer to that of a Spheroid similar to the Earth, than of a Sphere.

'Tis thus at least that Sir *Isaac Newton* *, the Inventor of the *Hypothesis*, seems to understand it. Speaking of this *Mass*, he uses the Words *paulo densior*; and, which Dr. *Gregory* (*Prop. 52. lib. III.*) seems to have overlooked; he measures its Attraction not from the Centre, as he might have done, but from *the Matter itself*; on purpose to insinuate that he thought it was (at least nearly) similar in Figure to the Earth.

If it be asked, how such a Spheroid could itself acquire its Figure, seeing the centrifugal Forces were not equal to that Effect? I answer, That if the ambient Fluid had been perfectly homoge-

* *Hæc ita se habent ex hypothesi quod Terra ex uniformi materia constet. Nam se materia ad centrum paulo densior sit quam ad superficiem, differentia Pendubrum & graduum Meridiani paulo majores erunt quam pro tabulâ præcedente, propterea quod si materia ad centrum redundans quâ densitas ibi major redditur, subducatur & seorsim spectetur, gravitas in Terram reliquam uniformiter densam erit reciproce ut distantia ponderis a centro; in materiam vero redundantem reciproce ut quadratum distantia a materiâ illâ quam proxime. Gravitas igitur sub æquatore minor est in materiam illam redundantem quam pro computo superiore; & propterea Terra ibi, propter defectum gravitatis, paulo altius ascendet & excessus longitudinum pendulorum & graduum ad polos paullo majores erunt quam in præcedentibus definitum est. Prop. 20. Lib. III.*

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neous, it might be impossible to account for it. But if we consider, that in the *Chaotic* state, this Fluid was undoubtedly the most heterogeneous that can be imagined, we shall, in shewing how the central Mass *might* acquire the Figure of a Spheroid, find a good Argument why it *could* not well receive any other.

IN that heterogeneous Fluid, one Effect of its Revolution must have been to dispose its Particles *in equilibrio* amongst themselves. By which means, the more dense must have taken place nearer the Equator, where some small part of their Weight being sustained by the centrifugal Force, they might be reduced to an equal relative Gravity with others nearer the Pole. From this excess of Density in the Fluid nearer the Equator, would follow a stronger Attraction to the central Mass, which is at first supposed spherical, and consequently a more easy and copious *Accretion* to it, in the Form of a solid Body. And the Equator of the central Mass being once raised sensibly higher, the Cause of the greater Density of the Fluid there, would likewise be a little augmented; because now the Weight of a Particle would be diminished, not only by the centrifugal Force, but by its Distance from the Centre of the Spheroid, (by *Cor. 2. Prop. 91. Lib. I. Princip.*) And this Process, we may suppose, went on till the Fluid was purged of all the denser Matter, that would adhere to the central Mass, that is, till it was interrupted by the contiguity of some Matter, less disposed to quit its fluid state.

SUCH may have been the Formation of a central Body denser than the surrounding Earth, and perhaps never firmly attach'd to it. Whose Existence seems necessary not only to the Solution of the Phenomenon in question, but of another that is of much greater Consequence. For one cannot help imagining it to be the same with that *Nucleus*, by which Dr. *Halley* so ingeniously explains the *Magnetic Variation* (*Phil. Transact. N° 195.*) If this should be the case, and if the determining its Magnitude is of any use in fixing the *Law* of that *Variation*, then shall this Enquiry be found less *idly speculative* than, to most Readers, it may at present appear.

III.

NOTWITHSTANDING the general Agreement that had been found between Theory and Experience to give the Earth the Form of an oblate Spheroid, the Obstinacy of some foreign Mathematicians render'd it necessary that this matter should be further inquired into, and in a different Manner than had yet been used:

MR. CASSINI, in tracing the Meridian of *France* from *Dunkirk* to *Collioure* in *Roussillon*, had computed that a Degree of the Meridian lengthens in going Southward, so as to make the Earth a *Pro-late* Spheroid higher at the Poles by about 95 Miles. Sir *Isaac* remonstrated *, “ That if so, Bodies must be lighter, and a Pendulum longer at the Equator than in *France* by about half an Inch; “ and that the Diameter of the Earth’s Shadow from South to “ North, must exceed its Diameter from East to West by 2'. 46", “ a twelfth part of the Moon’s Diameter. All which was manifestly “ contrary to Fact and Observation.” But this could not avail with Persons who were disposed to cavil.

FRANCE was chiefly the Seat of this Dispute; where although there are Numbers of Gentlemen too well acquainted with Sir *Isaac Newton*’s Reasonings, not to allow them their due Weight, and too sincerely attached to Truth to be influenced by any Prejudice or Authority; yet some, overawed perhaps by the Opinion of two or three noted Mathematicians of another Country, and supported by the Observations of so diligent an Artist as Mr. *Cassini* is allowed to be, could not be satisfied with any Reason or Experiment that had been adduced. At length the KING, from a Care of the Sciences, and at an Expence, becoming so great a Monarch, sent out two Companies of Mathematicians, one to *Peru*, the other as far North as they could go; who by actually measuring a Degree might decide this Question concerning the Figure of the Earth, as being not only a matter of Curiosity, but of some importance in Navigation, and other practical Arts.

FOR

* *Ibid.*

FOR the better understanding both the Ground and the Determination of this Dispute, the Reader is desired to recollect what hath been already explained concerning the Method of Measuring an Arc of a Meridian Circle. There he will easily perceive, that (in *Fig. I.*) if the Semidiameter CA had been less, the Arc AB answering to any given Difference of Altitude of the Star S, would have been diminished proportionally; and the contrary, if CA had been greater. Now 'tis plain, that a small Arc of the Ellipse PAPE (*Fig. V.*) may be considered as part of the Circumference of a Circle; and that the Semidiameter of this Circle will be least, if the small Arc is taken at A, the Vertex of the great Axis; greatest, if it is taken at P; and any where else, of an intermediate Magnitude, greater or less as it approaches *this* or *that* Extreme. If therefore, as Mr. *Cassini* formerly thought†, a Degree lengthens in going towards the Equator, the Semidiameters of the *Equicurve* Circles must increase likewise, that is, the Equator will be at P, *p*, and the Poles at A, E. But the Gentlemen who went Northward by the *French King's* Order, assure us, that just the Reverse of this is true; that the Arc of a Terrestrial Meridian at the Polar Circle, answering to one Degree in the Heavens, exceeds the like Arc in *France* by a very sensible Difference.

How safely the Observations of these Gentlemen may be relied on, appears from the above-cited Treatise of Mr. *de Maupertuis*, who was at the Head of this Commission. I shall mention some of the principal Circumstances, which render their Observations less exceptionable, than any of that kind that have been made. As,

I. IN the Astronomical Part, they used a Sector which could be verified to the Difference of one Second; and with which they could observe to a Degree of Exactness altogether incredible; if we did not know that this Instrument was contrived on purpose, and the Limb divided by Mr. *Graham*.

THE

† He has, I am told, of late ingenuously owned his Mistake.

INTRODUCTION.

2. THE fixt Stars which they observed (δ and α *Draconis*) were so near their Zenith, that there can be no Suspicion of any Error from the Refraction of Light.

3. THE Difference of Altitude was corrected not only by the known Equation of the Præcession of the Equinoxes, but by that new Equation discovered and most ingeniously accounted for by Mr. *Bradley*, in *Phil. Trans.* N^o 406.

4. UPON repeating their Observations, with the Star α , there was a Difference only of two Seconds, from the Angle they had found by the Star δ .

5. IN the *surveying* Part; the *Base* which they measured was in a Surface the fittest for their purpose that can be imagined; the Ice of a River which at that Place forms it self into a sort of Lake,

6. THIS *Base* was measured with Poles precisely of the same length, by two distinct Companies, who perfectly agreed in their Numbers.

7. MR. *DE MAUPERTUIS*, in a Discourse read to the *Royal Academy of Sciences*, shews, that in this Operation the just Length of the *Base* that is measured, depends upon the degree of Exactness, with which the Difference of Altitude can be observed, compared to that with which the Horizontal Angles can be taken. And applying this Theory to his *Base* at the Polar Circle, he finds it was of that *just* length.

8. THE Angles between the Signals were observed with a Quadrant of two Foot Radius, furnished with a Micrometer. The Signals themselves were carefully reduced to the same Horizon. And the Passages of the Sun by the Verticals of two of their Signals, from which the Position of the Polygonal Figure was determined, were observed with an Instrument consisting of a Telescope, and a Hori-

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Horizontal Axis upon which it moved in a vertical Circle; the Moments of these Transits being marked upon a Clock of Mr. *Graham*. The Instrument too was Mr. *Graham*'s Contrivance.

9. THE Number and Position of the Signals answered the Conditions *Pag. xiii*. They formed a Heptagonal Figure within which the Base lay, and furnished a great many Combinations of Triangles, from which the Arc of the Meridian, severally computed, was always nearly of the same Length. They put the Case, that in each of these Triangles, they had mistaken 20" in two of the Angles, and 40" in the Third, and that all these Errors contributed to shorten the Arc of the Meridian. And even upon this strange Supposition, there arose a Difference only of $54\frac{1}{2}$ Toises.

IF to all this we add the known Skill and indefatigable Industry of the Persons who were employed in this Affair; and that they were six in Number, who wrote down each his Observation apart, of all which, when there happened any small Difference, the *Mean* was taken; we must own that nothing better can be expected, or scarce wished for, on this Subject.

THE Result of their Operations was, "*That the Degree of the Terrestrial Meridian which cuts the Polar Circle, contains 57437.9 Toises; exceeding a Degree between Paris and Amiens, as measured by Mr. Picart, (after proper Allowances for the Refraction of Light, the Precession of the Equinoxes and Mr. Bradley's Equation) by 512.2 Toises.*". And upon this Determination the following Tables are calculated.

I must not however omit mentioning, that, since my writing this Paper, Mr. *de Maupertuis*, upon examining with Mr. *Graham*'s Sector, the Difference of Altitude observed by Mr. *Picart*, finds it is less by a few Seconds. But it was needless for this to make any Alteration in the Tables; especially seeing if ever it is thought proper to publish them in another Form, fitted for common Use, they must, by the Rules I have given, be calculated anew, or corrected, upon the best Observations that shall have then been made.

POST-

INTRODUCTION. POSTSCRIPT.

THE Series given by Mr. *de Maupertuis* for determining the Ratio of the Semidiameter of the Equator to the Semiaxis, is this,

$$\frac{E \times 1 + \frac{3}{2} \times m^2 - 1 \times S^2 + \frac{3}{8} \times m^2 - 1^2 \times S^4 + \&c.}{F \times 1 + \frac{3}{2} \times m^2 - 1 \times s^2 + \frac{3}{8} \times m^2 - 1^2 \times s^4 + \&c.} =$$

In which E and F are the Lengths of the two Degrees measured, S and s the Series of the Latitudes respectively, to the Radius 1; the Semiaxis *m*, and the Radius of the Equator Unity. From which,

putting D for $1-m$ he draws the Formula's $D = \frac{E-F}{3 \times ES^2 - Fs^2}$ or

$$D = \frac{E-F}{3E \times S^2 - s^2}.$$

ERRATA.

Pag. xii. Line 1. dele 1^o. Pag. xxii. l. penult. draw a Line over $c+b$. And over $1-r^2$ lin. 10. pag. xxiii. *Ibid.* for x^2 put z^2 . Pag. xxiv. in the Note, for $\frac{1}{360}$ read $\frac{1}{360}$.

In Fig. V. supply an inscribed Circle cutting AE in Q, q.

MER-



MERCATOR's SAILING,

Applied to the

True FIGURE of the EARTH.

THE Improvement of *Navigation* being one professed Design of the late painful and expensive Undertaking of the *French Academicians*, I thought it might be worth while to examine how far the *Spheroidal* Figure of the Earth, resulting from their Observations made at the Polar Circle, really affects the Art of Sailing; and if the Tables and Charts now in use, shall need any sensible Correction.

Plain Sailing, 'tis known, has no Place in long Voyages, because it does not answer to a Round Figure. *Great Circle Sailing* gives the shortest Distances of Places on a Sphere, but not their Bearings, nor can agree to any Course that a Ship steers by the Compass, unless she sails in a Meridian, or in the Equator it self. And although, by first gaining the Latitude of the Place to which a Ship is bound, and then keeping exactly in the same Parallel, a Port whose Situation Eastward or Westward is known, might be found with great certainty; yet this Method is both tedious and indirect. The only true Sailing, to which Tables and Charts must be adapted,

2 MERCATOR'S SAILING

being that upon a *Rumb-Line*, which cuts every Meridian at the same Angle.

BUT as Sea-Charts ought to be reduced to the simplest Form possible, that is, ought to be projected by Strait Lines only; in such a Projection, if the Degrees of the Equator and its Parallels, are made all equal to each other, those of the Meridian, though they were in themselves equal, must be represented by Lines continually increasing from the Equator to the Pole.

THE History of this useful Invention, with the several Improvements it had received, may be seen in the *Philosophical Transactions* N^o 219. where Dr. Halley gives the genuine and most expeditious Method of constructing *Mercator's* (or rather Mr. Wright's) *Table of Meridional Parts*; deducing it, in his Masterly Way, from the Property of the *Logarithmic Spiral*, and shewing that the *Nautical Meridian Line* is a Scale of *Logarithmic Tangents of the half Complements of the Latitudes*. Mr. Cotes demonstrates the same, by his Method of *Ratios*. (*Harm. Mensur.* p. 20, 21.)

AND Mr. George Campbell, in a Manuscript which I have seen several Years ago, laying aside the *Logarithmic Spiral*, uses only the following LEMMA.

LEMMA. FIG. I.

Let $QL (=c)$ be a Circular Arc, whose Complement is LP ; let $LS (=s)$ be its Cosine; and LT , PT , Tangents at L and P , meeting in T , either of which (t) will be the Tangent of $\frac{1}{2} LP$. Then I say, the Fluxion of the Arc QL , will be to the Fluxion of the Tangent t , as the Cosine LS is to the same Tangent.

DEMONSTRATION.

Let LS move into the Place ls , till the Arc QL is increased by the Quantity Ll ; at l draw the Tangent lt , intersecting PT , LT in r and x , so shall Tr be the Decrement of the Tangent PT , an-

Applied to the true Figure of the Earth.

3

swering to the Increment of the Arc Ll . And from x as a Centre, through T and l , Describe the Arcs Tv, lq , cutting lt, LT , in v and q . Draw also the Semidiameter KL , which put equal to Unity. Then will $x l : q l :: x T : Tv = \frac{ql \times xT}{xl}$. But if ls is supposed to move back into LS , the Triangle Ttv will at last be rectilinear, $Lx + xq$, that is, $2lx$ will be $= Ll$, or $c, lq = \frac{1}{2}c^2$ (by *Cor. 1. Lem. XI. Princ.*) and $xT = t$. Whence $Tv = \frac{\frac{1}{2}c^2 \times t}{\frac{1}{2}c} = c \times t$. But, by the then similar Triangles $LKS, Ttv, LK : LS$ (or $1 : s$) :: $Tt : Tv = s \times Tt$; and therefore $s \times t = t \times c$, or $c : t :: s : t$. *Q. E. D.*

PROBLEM I. FIG. 2.

Given ABC the Angle of a Ship's Course on a Sphere, with the Difference of Latitude $A\beta$, to find CK the Difference of Longitude.

SOLUTION.

LET PKZ be the Meridian, and AK the Latitude whence the Ship sets sail, βBL the Parallel where she is arrived; Br, br, Cc the Fluxions of that Parallel, of the Ship's present Latitude, and of the Longitude respectively, and the Semidiameter of the Sphere Unity. For the Cosine of the Latitude $K\beta$, that is, for the Sine of the Arc LP put s , and for the Tangent of $\frac{1}{2}LP$, write t . Then will Cc be to Br , as the Semidiameter of the Sphere is to the Semidiameter of the Parallel βBL , that is $\frac{Cc}{Br} = \frac{1}{s}$. And $Br : rb ::$

$n : m$ (in a given Ratio by *Hyp.*) Therefore $Cc = \frac{n}{m} \times \frac{rb}{s}$. But by the Lemma $\frac{rb}{s} = \frac{t}{t}$, whence $Cc = \frac{n}{m} \times \frac{t}{t}$. But if for the Tangent of half the Complement of the Latitude AK , from which the Ship sets out, we write T , then it is plain that when $CK = 0$,

4 MERCATOR'S SAILING

the Fluent of $\frac{n}{m} \times -\frac{t}{t}$ ought to vanish likewise. Whereas it is then $= \frac{n}{m} \times \text{Fl.} - \frac{T}{T}$. This therefore subtracted, CK will be $= \frac{n}{m} \times F \frac{T}{T} - F \frac{t}{t} =$ the Difference of the Hyperbolic Logarithms of T and t multiplied by $\frac{n}{m}$.

COROL. I.

IF A is at α (in the Equator) then $T = \text{Rad.}$ and the Difference of Longitude αC , will be $\frac{n}{m} \times \overline{l.R - l.t}$.

COROL. II.

SUPPOSE now the curvilinear Triangle αBC is to be protracted on a Plane (*Fig. 3.*) where a right Line CD divided into equal Parts, shall represent the Equator and its Degrees, and Perpendiculars to it (CP, QP, &c.) The Meridians (CBP, KAP, &c.) then as the Sine of αBC is to the Cosine, that is, $n : m :: \alpha C : CB = \frac{n}{m} \times \alpha C$.

But αC was shewn $= \frac{n}{m} \times \overline{l.R - l.t}$; Wherefore $CB = \overline{l.R - l.t}$.

Whence it is evident, that the Degrees of Latitude will be rightly marked on the Meridian, if a Ship is imagined to sail from the Equator at an Angle of 45° ; and if the Longitudes answering to the several Latitudes at which she arrives, are transferred on the Meridian Line; for $\overline{l.R - l.t}$ ($= CB$) is the Longitude, when $n = m$.

IN the same Paper Mr. *Campbel* gives a great many curious Theorems and Rules derived from this Solution, some of which are entirely new: But this is all I can recollect of it, and indeed all that is necessary for my present purpose.

PROBLEM

PROBLEM II.

LET Fig. 2. now represent a *Spheroid*, generated by the Revolution of an Ellipse round its lesser Axis PZ ; the Semidiameter of whose Equator, KE is Unity, and its Semiaxis $KP=a$. And let it be required to divide the nautical Meridian Line of this Spheroid, or which is the same thing, to find the Longitude αC belonging to any Latitude $K\beta$, in a Course of 45° .

SOLUTION.

LET LS , an Ordinate to the Axis from the Parallel where the Ship arrives be called z , and the Abscissa PS , y . Draw the Tangent LM , and put rb , or $Ll=c$, $Cc=v$; then the Equation to this Ellipse being $z^2 = \frac{2ay-y^2}{a^2}$; and $y=a-a \times \sqrt{1-z^2}$, it will very

easily appear, that the Subtangent $SM = \frac{az^2}{\sqrt{1-z^2}}$. But $c : -\dot{z} ::$

$$\sqrt{SM^2 + z^2} : z :: \frac{\sqrt{a^2 - 1 \times z^4 + z^2}}{\sqrt{1-z^2}} : z :: \frac{\sqrt{1+a^2-1 \times z^2}}{\sqrt{1-z^2}} : 1 :: (\text{putting } q = 1-a^2) :: \sqrt{1-qz^2} : \sqrt{1-z^2}; \text{ or } c = \sqrt{1-qz^2}^{\frac{1}{2}} \times$$

$\sqrt{1-z^2}^{-\frac{1}{2}} \times -\dot{z}$. But by a like reasoning as in *Prob. 1.* $\dot{v} = \frac{c}{z}$. If therefore, for c there be substituted its value just now found, express'd in a Series, and the Fluents be taken, putting likewise l for a hyperbolic Logarithm, we shall have $v (= \alpha C) =$

$A - l z - \frac{1}{4} z^2 - \frac{3}{32} z^4 - \frac{5}{96} z^6 - \frac{35}{1024} z^8 - \frac{63}{2560} z^{10} - \frac{231}{12288} z^{12} - \dots$

$$\begin{aligned} & + \frac{1}{4} q + \frac{1}{16} q + \frac{3}{96} q + \frac{5}{256} q + \frac{7}{312} q + \frac{21}{3072} q + \dots \\ & + \frac{1}{32} q^2 + \frac{1}{96} q^2 + \frac{3}{312} q^2 + \frac{1}{256} q^2 + \frac{35}{12288} q^2 + \dots \\ & + \frac{1}{96} q^3 + \frac{1}{256} q^3 + \frac{1}{3072} q^3 + \frac{3}{3072} q^3 + \dots \\ & + \frac{1}{1024} q^4 + \frac{1}{312} q^4 + \frac{5}{960} q^4 + \dots \\ & + \frac{7}{2560} q^5 + \frac{7}{6144} q^5 + \dots \\ & + \frac{7}{4096} q^6 + \dots \end{aligned}$$

$\dots$

Where

Where A stands for the Value of the Series $—l. z — \frac{1}{2} + \frac{1}{2} q \times z^2 \&c.$ when z becomes equal to the Semidiameter of the Equator, with the Signs changed.

COROL. I.

IF $q=1$, i. e. if the Figure is a Sphere, the Series now wanting the Terms whose Coefficients q enters, will exhibit the common Divisions of the Meridian Line, and will be (*Cor. 2. Prob. 1.*) $=l. R — l. t.$

IF the Sign of q is changed, it will give the like Divisions for a Prolate Spheroid.

IF $q=-1$, $v=l. R — l. z$ is the Equation of a logarithmic Spiral, described on the Plane of the Equator.

AND, in General, the same Method may be applied to any Figure generated by the Revolution of a Curve about an Axis.

COROL. II.

SETTING aside $l. z$, for all the following Terms in the same Line write g , and for the remaining Terms of the Series, which are affected with q and its Powers, put b . For the same things put $l. R$, G and H , when $z=1$; then will $v=l. R + G — H — l. z — g + b$. But (by *Cor. 1.*) if t = the Tangent of $\frac{1}{2}$ the circular Arc whereof z is the Sine, then will $l. R + G — l. z — g = l. R — l. t$; or $l. z + g — l. t — G = 0$; which added to the foregoing Equation, gives $v=l. R — l. t — H — b$.

FROM these Corollaries a *Table of Meridional Parts* for the Spheroid may be calculated by the following Rules.

I.
FROM the Observations that have been made, find the Species of the Spheroid, *i. e.* the Value of q ; which, according to Monf^r. de Maupertuis's Measures, will be nearly .022.

II.
FOR any assumed Latitude LMS (*Fig. 4.*) whose Sine and Co-fine, to the Radius 1, are S and Σ , find the Ordinate LS = z ; which will always be $\sqrt{\frac{\Sigma}{1-q^2}}$.

III.
APPLY, in the circumscribed Circle Q λ π E, $\lambda\sigma t$ equal and parallel to LST, and by *Prob. 1.* that is, by Dr. Halley's Rule, find the Meridional Parts for the Latitude Q λ , or for the Angle $\lambda m\sigma$ (which in an Oblate Spheroid will always be less than LMS) and from these Parts subtract H— b , reduced to the same Denomination of Minutes, or of tenths, &c. of Minutes.

IV.
STILL I ought to set down the Method of computing the H— b ; for in the common way of summing up the Coefficients that stand over each other, and multiplying them into the respective Powers of z , it would be quite impracticable, the numeral Coefficients of q , q^2 , &c. converging so slowly. Besides, it would be necessary when z approaches to Unity, to use another Series.

I consider therefore any part of H— b that is affected with the same Power of q , as the Area of a Curve. And it happens, in the present Case, that all these Areas are assignable in finite Terms; and that q being so small, two of them, *viz.* which belong to q and q^2 are almost always a sufficient Approximation. To find, for Example, that Part of H— b which is affected with q^3 ; I write down the Fractions $-\frac{1}{3} + \frac{2}{3} - \frac{1}{3}$, whose Numerators are the Unciæ of $\overline{a+b}^{3-1}$, and the Denominators the odd Numbers in an inverted Order; the Signs being alternately $\pm$, $\mp$, as the Index of q is even or

or odd. To these Coefficients I affix $x^{\frac{1}{2}}, x^{\frac{3}{2}}, x^{\frac{5}{2}},$ (having made $x=1-z^2$) where the Numerators of the Indices are the Denominators of the Coefficients, and the constant Denominator 2. And I have $-\frac{1}{3}x^{\frac{1}{2}}+\frac{1}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}},$ which multiplied into $-\frac{1}{16}q^3,$ the Coefficient of that Term of $1-qz^2)^{\frac{1}{2}}$ from which the Part required was produced, gives $-\frac{1}{16}q^3 \times -\frac{1}{3}x^{\frac{1}{2}}+\frac{1}{3}x^{\frac{3}{2}}-1 \times x^{\frac{5}{2}}$ for the Area sought. In like manner the Area belonging to q is $-\frac{1}{2}q \times -\frac{1}{4}x^{\frac{1}{2}}+0=\frac{1}{8}qx^{\frac{1}{2}};$ that of q^2 is $-\frac{1}{8}q^2 \times \frac{1}{3}x^{\frac{1}{2}}-\frac{1}{4}x^{\frac{3}{2}}=-\frac{1}{8}q^2 \times \frac{1}{3}x^{\frac{1}{2}}-1 \times x^{\frac{3}{2}};$ that of $q^3,$ $-\frac{1}{256}q^3 \times -\frac{1}{9}x^{\frac{1}{2}}+\frac{1}{7}x^{\frac{3}{2}}-\frac{6}{5}x^{\frac{5}{2}}+\frac{4}{3}x^{\frac{7}{2}}-1 \times x^{\frac{9}{2}},$ &c.

N. B. When z is small, if it should be thought convenient to calculate b separately, it may be observed that H is equal to the Series $\frac{1}{16}q+\frac{1}{32}q^2+\frac{1}{56}q^3+\frac{1}{72}q^4-\frac{1}{2n-1 \times 2n} \times q^n=.011040692.$

OR if, in the same Case, one would calculate the meridional Parts immediately from the Series, $G(=\frac{1}{4}+\frac{1}{32}+\frac{1}{96}+\&c.)=.693147.$

Construction

Construction of TABLE III.

FOR QP (Fig. 4.) the Quadrant of the Elliptical Meridian write Q. And let LS ($=z$) be the Semidiameter of any Parallel of Latitude; then, from the Solution of Prob. II. it will easily appear, that the Arc

$$QL = Q - z - \frac{1}{6}z^3 - \frac{3}{40}z^5 - \frac{5}{112}z^7 - \frac{35}{1152}z^9 - \left. \begin{array}{l} + \frac{1}{6}q + \frac{1}{10}q + \frac{1}{112}q + \frac{5}{1152}q + \\ + \frac{1}{40}q^3 + \frac{1}{112}q^3 + \frac{3}{370}q^3 + \\ + \frac{1}{112}q^3 + \frac{1}{1152}q^3 + \\ + \frac{1}{1152}q^4 + \end{array} \right\} \text{ &c.}$$

BUT here, for the like Reasons as in the former Solution, it will be convenient to sum this Series as follows.

I.

THE first Line, setting aside Q, is the circular Arc whereof the Sine is z , to the Radius Unity; call this Arc A.

II.

FOR the Cosine of the same Arc write y , then will the second Line (i. e. $\frac{1}{2}q \times F : z^2 \times 1 - z^2 - \frac{1}{2}z^4$) be $\frac{1}{2}q \times \frac{1}{2}A - \frac{1}{2}zy (= \frac{1}{2}q \times B.)$

III.

THE third Line $\frac{1}{8}q^2 \times \frac{3}{4}B - \frac{1}{4}z^3y (= \frac{1}{8}q^2 \times C.)$

AND the 4th is $\frac{1}{8}q^2 \times \frac{5}{8}C - \frac{1}{8}z^3y$, &c. All which is plain from Mr. de Moivre's 4th Theorem of Quadratures in *Philosophical Transactions* N^o 278. And thus the Arcs QL may be computed with all imaginable Ease, remembering only that the Parts expressed in z and y , are to be reduced to the same Denomination with A; which, if A is expressed in Minutes, is done by multiplying them into 3437.44675.

IV.

Q, that is the whole Elliptical Quadrant QP, is, in this Method found at once; for A is now the Quadrant of a Circle, and the Products of z, y , vanish. Whence $Q =$

$$\frac{A - \frac{1}{2}q \times \frac{1}{2}A + \frac{1}{8}q^2 \times \frac{3}{4}B + \frac{1}{16}q^3 \times \frac{5}{8}C + \text{&c.}}{A \times 1 - \frac{1}{2}q \times \frac{1}{2} - \frac{1}{8}q^2 \times \frac{1}{2} \times \frac{3}{4} - \frac{1}{16}q^3 \times \frac{1}{2} \times \frac{3}{4} \times \frac{5}{8} + \text{&c.}}$$

V.

OR if you would compute QL independent of Q, which is convenient when QL is small, the same Expressions will still serve; observing only, *First*, That z and y now change Places, v. g. The third Line is now $\frac{1}{8}q^2 \times \frac{3}{4}B - \frac{1}{4}y^3z$, and so of the rest. *Secondly*, That the Sum of the Parts affected by q is not to be subtracted from A, but added to it. *Thirdly*, That this last Sum is to be diminished in the Ratio of the greater Axis to the lesser.

SCHOL. I.

IF $q = 1$, the Arc QP degenerates into the Semidiameter QK, and the Equation in N^o IV becomes

$$Q = A \times \frac{1 - \frac{1}{2} \times \frac{1}{2} - \frac{1}{8} \times \frac{1}{2} \times \frac{3}{4} - \frac{1}{16} \times \frac{1}{2} \times \frac{3}{4} \times \frac{5}{8} - \text{&c.}}{1}$$

$$\text{Or } Q : A :: 1 - \frac{1}{4} - \frac{1.3}{4.4} \alpha + \frac{3.5}{6.6} \beta + \frac{5.7}{8.8} \gamma + \text{&c.} : 1.$$

But the Series $\frac{1}{4} + \frac{1.3}{4.4} \alpha + \frac{3.5}{6.6} \beta + \text{&c.}$ summed by the Scholium of the 11th Proposition of Mr. Sterling's *Meth. Differ.* is $= .3633803$, and $1 - .3633803$ (i. e. 6366197) : $1 :: 1 : 1.570796 :: Q : A$; that is, as the Semidiameter of a Circle is to the 4th Part of its Circumference.

SCHOL. II.

IN calculating these Tables, I have put q , that is, the Difference of the Squares of the Semiaxes, equal to .022; which by the best Surveys

Surveys that have been made, must be very near the Truth. * But if there were any Survey, especially near the Equator, of equal Accuracy with that of Mon^r. *de Maupertuis*, to which his could be compared, then q might be more precisely determined. And in order to this, his Rule (Fig. of the Earth *p.* 167.) would scarce be sufficiently exact. The two small Errors which his Methods involve, would then discover themselves; One, That the Degree in the Middle of the Arc measured, is to be rated by the Length of the whole; the other, That this Degree is an Arc of a Circle. The Way of proceeding would then be, *First*, To approximate to q by Mon^r. *de Maupertuis*'s Series, (taking in, if you will, another Term of it.) And then in the Spheroid thus determined, to compute (as in the Construction of Tab. III.) the Lengths of the whole Arcs measured; noting the Differences between the computed and measured Arcs. And having repeated the like Operation with another q , somewhat nearer the Truth, you might from these Differences find, by the *Rule of Position*, a q that would give the computed and measured Arcs of the same Length.

So much for the *Construction*; it remains now to shew

* Figure of the Earth determin'd, *pag.* 162, 163. *Lib.* 3. *Prop.* 18, 19, 20. *Princ.* Newton.



The USE of the TABLES.

IF a Ship's Course is all in the same Meridian, Questions concerning it are solved by Inspection of Tab. III. or at most by Subtraction. And it is only to be noted,

First, THAT the Total Difference between two Arcs of the Sphere and Spheroid, reckoned from the Equator to the same Latitude, is greatest at near 55° ; because there the Radius of Curvature is equal to the Semidiameter of the Equator. Afterwards this Difference decreases, till at the Pole it ends in somewhat less than 30 Minutes.

Secondly, THAT yet the Ratio of this Difference to the whole Course, is perpetually decreasing from the Equator; where it is for the first Degree $\frac{1}{317}$, at 55° $\frac{2}{817}$, and at the Pole no more than $\frac{1}{31701}$.

Thirdly, Courses, the Differences of whose extreme Points are equidistant from the greatest Difference, are the same as if performed on the Sphere. As in sailing from 45° to 65° : If one Extreme is nearer the Pole, the Course is lengthned; if towards the Equator, it is shortned.

TAB. III. must likewise be used in the Reduction of the rectilinear *Rumb-Lines* on the Chart, to the *Distances run*.

II.

IN East or West Courses, say, As Rad. to the Semidiameter of the Parallel (in Tab. II.) so are the Minutes in the Difference of Longitude to the Distance run. Or if the Distance is given, say, As the Semidiameter of the Parallel to Rad. so is the Distance to the Difference of Longitude.

Applied to the true Figure of the Earth. 13

RESUMING the Symbols z, s, Σ, q , and putting D for the Distance sailed on a Parallel of the Spheroid, and d for that sailed on the same Parallel of the Sphere, the common Difference of Longitude being L , it will be $z : \Sigma :: D : d$, and $z - \Sigma : z :: D - d : D$. But because it was $z = \frac{\Sigma}{\sqrt{1-q^2}}$, the Ratio of $z - \Sigma$ to z will be $1 - \sqrt{1-q^2}$; and this when greatest, *i. e.* when $s = 1$, is $1 - \sqrt{.978} = .01106$. So that $D - d$ can never much exceed a hundredth Part of the whole Distance run. In like manner it is shewn, that, to the same Distance sailed on a Parallel of the Sphere or Spheroid, the Differences of Longitude cannot differ by much above $\frac{1}{100}$ of that belonging to the Spheroid.

III.

To find how the Spheroidal Figure of the Earth affects *oblique Courses*, from Tab. I.

LET EQ (in Fig. 5.) represent the Equator of the nautical Chart, or any Parallel common to the Spheroid and Sphere; KH any other Parallel of the Sphere, CG the same Parallel of the Spheroid, EK a Meridian, and CK the Difference of meridional Parts. Bisect CK in M, from which raise MR perpendicular to it; and from C, at the Distance ME describe an Arc intersecting MR in R. Then will R be the Centre of a Circle KCA, which passes through K, C, and touches EQ in A; and the Angle CAK will be greater than any other CBK made by Lines drawn from C, K to a Point B in EQ that is not the Point of Contact. That is, *AE thus determined, is the Difference of Longitude of two Places A and C, which ought to be assumed to make the Difference of the Angles of the Courses on the Spheroid and Sphere the greatest possible.* But because CK is inconsiderable in respect of CE, AE may be put = EM. Thus if EQ is the Equator, and CG the Parallel of 45° , AE will be = 3003, and the Angle CAK will be found to be about $30\frac{1}{2}$; greater than it would have been upon the Supposal of any other Difference of Longitude EB.

To compare the Angle CAK with another (*ca k*) between the Equator and any other Parallel; from A as a Centre through C describe the Arc CN, meeting AK in N; and it is plain the Angle CAK will be proportional to $\frac{CN}{CA}$. But because so small an Arc may be considered as a right Line, and because AKE, ACE are not much different from half right Angles, $\frac{CN}{CA}$ will be nearly as $\frac{CK}{CE}$. And seeing $\frac{CK}{CE}$, in its greatest Magnitude at the Equator, is only $\frac{34}{978}$, *i. e.* the Ratio of the Difference of the Semidiameter of the Equator; and of the Radius of Curvature, to that Radius of Curvature it follows, that the Difference of the Angles of the Course, which at 45° was $30\frac{2}{3}'$, can never rise to above $38'$.

IN the same Figure KN (*Kn*) will represent the Difference of the Distance sailed, and will be to the Difference of a Course that lies all in the same Meridian, nearly as the Sine of KAE (*KBE*) to Radius.

CONCLUSION.

FROM all which it appears, * that the Errors in Navigation arising from the supposed spherical Figure of the Earth, are not so considerable as might have been apprehended; yet, small as they are, I believe no good Reason can be given why they ought not be corrected. In the mixt Sciences, one Part ought not to lose the Accuracy it can have, because the other Parts are incapable of quite so much. However difficult it is to take an Altitude at Sea, or steer a Ship's Course, yet a Sea-Quadrant would hardly be reckoned good, whose Divisions were generally wrong by 10 or 20 Minutes, sometimes by half a Degree; nor a Chart in which the Latitudes and Bearings of Places were so much different from what they ought to be, and whose Length should perhaps be an Inch too much in proportion to its Breadth; yet the Errors in the Tables and

* See the Preface to *Maupertuis's* Figure of the Earth determined.

Charts now used, are equivalent to those. Nor is it a good Answer to this, that in the Practice of Sailing, greater Errors still are unavoidable. For if a Ship's Course is at all regulated by Tables and Charts, whatever these *unavoidable Errors* are, they will happen sometimes to be increased by the *whole Error* of the Chart; and the Sum of two Errors may be fatal, where one Error alone would not have been so.



Some

Some other CONSEQUENCES of the Earth's Spheroidal Figure.

I.

IN Matters purely Geographical, the Earth may be consider'd as a perfect Sphere*; for Instance, in the common Stereographic Projection of the Globe, where it is only required that the Longitudes and Latitudes of Places be justly marked, it would be an idle Refinement and a spoiling the Elegance of the Projection, to substitute Ellipses for Circles, although the Difference of the Earth's Diameters was much greater than it is.

BUT in that Orthographic Projection which is made, or supposed to be made, for the easier Calculation of the Moments of Solar Eclipses, the Case is perhaps different. Were the Theory of the Moon compleated, the Differences of these Moments (of Beginning, End, or Total Obscuration) on the Sphere and Spheroid, might be, in some Circumstances, not imperceptible.

ASTRONOMERS will likewise consider how far the spheroidal Figure of the Earth may affect the Determination of the Sun's *Parallax* from the *Transit of Venus* in the Year 1761.

II.

THE *Parallax* of the Moon, at a given Distance from the Center of the Earth, will be greatest at the Equator, least at the Pole: And in any given Latitude, as at L (Fig. 4.) if perpendicular to LC, the Radius of Curvature of the Meridian, there is drawn through the Center the right Line AB, intersecting it in C; the Moon's *Parallax* at that Latitude will be to her *Parallax* at the

* Newton *Princ. Lib. III. Prop. 20.*

Equator,

Equator, as LC to QK. And in Lunar Eclipses, one Diameter of the Earth's Shadow will be diminished in a Ratio, easily assignable from the Sun's Declination at the time of the Eclipse.

III.

THERE results likewise from the Earth's spheroidal Figure, a small *Parallax* of the Moon's *Azimuth* and *Right Ascension*.

IN Fig. 4. Let LG represent the Intersection of the *Prime Vertical* of the Place L, with the Plane of the Meridian; and XY the like Intersection of a Plane passing through the Centre of the Earth, and parallel to the former. This last we may call the *Rational Prime Vertical*, and the former the *Sensible*. Now, because not only a Semidiameter of the Earth, but even CK the Distance of the *Prime Verticals*, bears some sensible Proportion to the Moon's Distance from the Centre K; if we suppose the Moon's apparent Place to be due West, *i. e.* That her Centre is in the *Sensible Prime Vertical*, some short Time must intervene before it is in the *Rational Vertical*. And by the same Time will the Moon's Centre have past the *Rational Prime Vertical* on the East, ere it arrives at the *Sensible*. But from the Species of the Spheroid, and the Latitude of the Place given, the Ratio of LC to CK is given also; say therefore, As LC to CK, so is the Moon's *Horizontal Parallax* to that of her *Azimuth*.

THUS if we put the Latitude of the Place 45° , and the Moon's horizontal Parallax $57\frac{1}{2}''$, or $3450''$, CL will be $=9944846$, and CK $=110610$, and the Parallax of the Azimuth $38''$.

OR if this Parallax is to be computed in *Right Ascension*: Supposing, first, that the Moon has no *Declination*, say, As LC to QK, that is in the present Case, as 9944846 to 10000000 :: so is $3450''$: $3469'' =$ the Moon's horizontal Parallax at the Equator. And, as QK to KF (*i. e.* 10000000 : 156426) :: so $3469''$ to $54''$ the *Parallax in Right Ascension* required; which is in Time $3''.6$.

IF the Moon has Declination towards the elevated Pole, compute what Part of her diurnal Arc is intercepted by the two vertical Planes produced : For this will be the parallax of right Ascension required.

IN other Verticals, the same Rule is to be observed, only it is to be remembred, that the Distance of the rational and sensible Verticals, which is the Measure of the azimuthal Parallax, decreases as the Sine of their Inclination to the Meridian, and in the Meridian itself vanishes.

S C H O L I

IF the Moon's Place could be exactly calculated and observed, the true Figure of the Earth might receive a new Proof from Astronomical Observation. And much more easily might the Opinion of Mr. *Cassini* and his Friends be confuted. For as they imagine the Earth to rise more than twice as high at the Poles, as it really does at the Equator, we may suppose in gross (for I have not calculated it) that this would give a *Parallax* about double to that which we have determined; and which might sometimes rise to above 2' in right Ascension, or 8' in Time, of a contrary Denomination; because the Sensible Vertical would now fall on the other side of the Rational.

IV.

I might add that there are, strictly speaking, no Antipodes, except at the Equator, and at the Poles; if there were, their erect Posture must be GL: But the erect Posture at G is Gg, therefore, &c. The Posture directly contrary to that at L, will be found at the other Extremity of the Diameter through L.

(19)

T A B. I. Of Meridional Parts to the Spheroid and Sphere, with their Differences.

D.	Spheroid.	Sphere.	Diff.	D.	Spheroid.	Sphere.	Diff.
1	58.7	60.0	1.3	17	1013.1	1035.3	22.2
2	117.3	120.0	2.7	18	1074.8	1098.3	23.5
3	176.1	180.1	4.0	19	1136.8	1161.6	24.8
4	234.9	240.2	5.3	20	1199.2	1225.2	26.0
5	293.8	300.4	6.6	21	1262.0	1289.2	27.2
6	352.7	360.6	7.9	22	1325.3	1353.7	28.4
7	411.8	421.0	9.2	23	1389.0	1418.6	29.6
8	471.0	481.5	10.5	24	1453.3	1484.1	30.8
9	530.4	542.2	11.8	25	1518.0	1550.0	32.0
10	589.9	603.0	13.1	26	1583.3	1616.5	33.2
11	649.7	664.1	14.4	27	1649.1	1683.5	34.4
12	709.6	725.3	15.7	28	1715.6	1751.2	35.6
13	769.8	786.8	17.0	29	1782.7	1819.5	36.8
14	830.2	848.5	18.3	30	1850.5	1888.4	37.9
15	890.9	910.5	19.6	31	1919.0	1958.0	39.0
16	951.8	972.7	20.9	32	1988.2	2028.3	40.1

Of Merid. Parts to the Spheroid and Sphere,

D.	Spheroid.	Sphere.	Diff.
33	2058.3	2099.5	41.2
34	2129.1	2171.4	42.3
35	2200.8	2244.2	43.4
36	2273.4	2317.9	44.5
37	2347.0	2392.6	45.6
38	2421.6	2468.3	46.7
39	2497.2	2544.9	47.7
40	2573.9	2622.6	48.7
41	2651.8	2701.5	49.7
42	2730.9	2781.6	50.7
43	2811.3	2863.0	51.7
44	2893.1	2945.8	52.7
45	2976.2	3029.9	53.7
46	3060.9	3115.5	54.6
47	3147.2	3202.7	55.5
48	3235.1	3291.5	56.4
49	3324.8	3382.1	57.3
50	3416.3	3474.5	58.2
51	3509.7	3568.8	59.1

D.	Spheroid.	Sphere.	Diff.
52	3605.3	3665.2	59.9
53	3703.1	3763.8	60.7
54	3803.1	3864.6	61.5
55	3905.7	3968.0	62.3
56	4010.9	4073.9	63.0
57	4118.9	4182.6	63.7
58	4229.8	4294.2	64.4
59	4344.0	4409.1	65.1
60	4461.5	4527.3	65.8
61	4582.7	4649.2	66.5
62	4707.8	4775.0	67.2
63	4837.1	4904.9	67.8
64	4971.0	5039.4	68.4
65	5109.8	5178.8	69.0
66	5254.0	5323.6	69.6
67	5403.9	5474.0	70.1
68	5560.2	5630.8	70.6
69	5723.5	5794.6	71.1
70	5894.4	5965.9	71.5

with their Differences.

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D.	Spheroid.	Sphere.	Diff.
71	6073.7	6145.6	71.9
72	6262.4	6334.7	72.3
73	6461.6	6534.3	72.7
74	6672.6	6745.7	73.1
75	6896.8	6970.3	73.5
76	7136.2	7210.0	73.8
77	7393.0	7467.1	74.1
78	7670.1	7744.5	74.4
79	7970.9	8045.6	74.7
80	8300.2	8375.2	75.0

D.	Spheroid.	Sphere.	Diff.
81	8663.8	8739.0	75.2
82	9070.0	9145.4	75.4
83	9530.2	9605.8	75.6
84	10061.1	10136.9	75.8
85	10688.7	10764.6	75.9
86	11456.5	11532.5	76.0
87	12446.0	12522.1	76.1
88	13840.4	13916.4	76.0
89	16223.8	16299.5	75.7
90	∞	∞	37.75



T A B.

T A B. II. Of the Semidiameters of the Parallels of Latitude on the Spheroid.

Semidiameter of the Parallel.
19998511
29994043
39986596
49976174
59962780
69946413
79927084
89904791
99879543
109851345
119820205
129786130
139749129
149709209
159666383
169620661

Semidiameter of the Parallel.
179572053
189520571
199466228
209409041
219349021
229286184
239220547
249152124
259080936
269007000
278930335
288850961
298768898
308684166
318596794
328506798

Semidiameter of the Parallel.
338414206
348319040
358221327
368121092
378018364
387913169
397805539
407695499
417583084
427468321
437351245
447231888
457110282
466986464
476860468
486732329

Of the Semid. of the Paral. of Lat. on the Spheroid. 23

D.	Semidiameter of the Parallel.
49	6602085
50	6469774
51	6335434
52	6199104
53	6060823
54	5920634
55	5778575
56	5634692
57	5489024
58	5341619
59	5192519
60	5041768
61	4889403
62	4735591

D.	Semidiameter of the Parallel.
63	4580078
64	4423193
65	4264894
66	4105227
67	3944247
68	3782000
69	3618539
70	3453915
71	3288179
72	3121383
73	2953580
74	2784825
75	2615169
76	2444669

D.	Semidiameter of the Parallel.
77	2273378
78	2101350
79	1928642
80	1755309
81	1581407
82	1406991
83	1232118
84	1056846
85	0881230
86	0705328
87	0529197
88	0352894
89	0176475
90	0

T A B.

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**T A B. III. Arcs of the Meridian to the Spheroid, in
Minutes of the Equator.**

D.	Spheroid.	Sphere.	Diff.
1	58.7	60.0	1.3
2	117.3	120.0	2.7
3	176.0	180.0	4.0
4	234.7	240.0	5.3
5	293.4	300.0	6.6
6	352.1	360.0	7.9
7	410.8	420.0	9.2
8	469.6	480.0	10.4
9	528.3	540.0	11.7
10	587.0	600.0	13.0
11	645.8	660.0	14.2
12	704.5	720.0	15.5
13	763.3	780.0	16.7
14	822.1	840.0	17.9
15	880.9	900.0	19.1
16	939.7	960.0	20.3

D.	Spheroid.	Sphere.	Diff.
17	998.5	1020.0	21.5
18	1057.4	1080.0	22.6
19	1116.3	1140.0	23.7
20	1175.2	1200.0	24.8
21	1234.1	1260.0	25.9
22	1293.0	1320.0	27.0
23	1352.0	1380.0	28.0
24	1411.0	1440.0	29.0
25	1470.0	1500.0	30.0
26	1529.0	1560.0	31.0
27	1588.1	1620.0	31.9
28	1647.2	1680.0	32.8
29	1706.3	1740.0	33.7
30	1765.5	1800.0	34.5
31	1824.7	1860.0	35.3
32	1883.9	1920.0	36.1

Arcs of the Merid. to the Spheroid, in Min. of the Equat. 25

D.	Spheroid.	Sphere.	Diff.
33	1943.1	1980.0	36.9
34	2002.4	2040.0	37.6
35	2061.7	2100.0	38.3
36	2121.0	2160.0	39.0
37	2180.4	2220.0	39.6
38	2239.8	2280.0	40.2
39	2299.2	2340.0	40.8
40	2358.7	2400.0	41.3
41	2418.2	2460.0	41.8
42	2477.7	2520.0	42.3
43	2537.3	2580.0	42.7
44	2596.8	2640.0	43.2
45	2656.6	2700.0	43.4
46	2716.4	2760.0	43.6
47	2776.2	2820.0	43.8
48	2835.9	2880.0	44.1
49	2895.5	2940.0	44.5
50	2955.3	3000.0	44.7
51	3015.2	3060.0	44.8

D.	Spheroid.	Sphere.	Diff.
52	3075.0	3120.0	44.0
53	3135.0	3180.0	45.0
54	3194.9	3240.0	45.1
55	3254.9	3300.0	45.1
56	3314.9	3360.0	45.1
57	3370.0	3420.0	45.0
58	3435.1	3480.0	44.9
59	3495.2	3540.0	44.8
60	3555.3	3600.0	44.7
61	3615.5	3660.0	44.5
62	3675.7	3720.0	44.3
63	3736.0	3780.0	44.0
64	3796.2	3840.0	43.8
65	3856.5	3900.0	43.5
66	3916.8	3960.0	43.2
67	3977.2	4020.0	42.8
68	4037.5	4080.0	42.5
69	4097.9	4140.0	42.1
70	4158.4	4200.0	41.6

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26 Arcs of the Merid. to the Spheroid, in Min. of the Equat.

D.	Spheroid.	Sphere.	Diff.
71	4218.8	4260.0	41.2
72	4279.3	4320.0	40.7
73	4339.8	4380.0	40.2
74	4400.3	4440.0	39.7
75	4460.8	4500.0	39.2
76	4521.3	4560.0	38.7
77	4581.9	4620.0	38.1
78	4642.5	4680.0	37.5
79	4703.1	4740.0	36.9
80	4763.7	4800.0	36.3

D.	Spheroid.	Sphere.	Diff.
81	4824.3	4860.0	35.7
82	4884.9	4920.0	35.1
83	4945.5	4980.0	34.5
84	5006.2	5040.0	33.8
85	5066.8	5100.0	33.2
86	5127.5	5160.0	32.5
87	5188.2	5220.0	31.8
88	5248.8	5280.0	31.2
89	5309.5	5340.0	30.5
90	5370.2	5400.0	29.8



ALTHOUGH

ALTHOUGH the Use of these Tables in Navigation has been above explained more generally, and, 'tis hoped, to the Satisfaction of the Mathematical Reader; yet for the sake of such as are concerned only in the practical Application, I have thought fit to add what follows.

I.

Of the Construction of a Nautical Chart to the Spheroid of the Earth.

THIS Construction differs from Mercator's (which is to be found in most Books of Navigation) in this only; that in laying down the *Latitudes* of Places, instead of his *Table* of Meridional Parts, *Tab. I.* is to be used.

THUS, in *Fig. 3.* If A, B are two places, whereof the first is at the *Equator*, the other in the *Latitude* 12° , (North or South) the Difference of *Longitude* (AC) being $5^{\circ}. 30'$; from any convenient Scale of equal Parts, each of which may represent one Minute of *Longitude*, I set off $AC=330$ the Number of Minutes in the given Difference of *Longitude*; and from C having raised CP perpendicular to AC, from the same Scale, I take in it $CB=709.6$, the meridional Parts for the *Lat.* 12° . (above or below AC, as the *Latitude* is North or South.) Then having joined A, B, the Line AB will represent the *Rumb-line* between the two given Places, and the Angle ABC their *Bearing*, or the Angle of the *Course* in Sailing from the one to the other.

IF A is in the *Latitude* 16° , B in the *Latitude* 27° , (both North) the Difference of *Longitude* being $5^{\circ}. 30'$ as before; from 1649.1 (the meridional Parts for the greater *Latitude* 27°) I subtract 951.8 (the meridional Parts for the lesser 16°) and make CB equal to their Difference 697.3. And AC is no longer the *Equator*, but the *Parallel* of 16° North *Latitude*.

OR *lastly*, If the same things are supposed as in the foregoing Case, with this Difference, that A is on the other Side of the *Equator*; then will AC be the *Parallel* of 16° South *Latitude*, and CB will now be equal to the Sum of 1649.1, and 951.8 that is to 2601.

IF, in these three Cases, the Lengths of CB had been found by *Mercator's Tables*, they would have been 725.3, 710.8, 2656.2; exceeding the former by 15.7, 13.5, 55.2 respectively.

IN the same manner from the *Longitudes* and *Latitudes* given, having marked every *Port*, *Island*, *Cape*, &c. which falls within the *Extent* of your *Chart*, the Consequences of such Construction will be,

I.

A Line AB joining any two Places, will make with the *Meridian* an Angle ABC, equal to a Ship's Course from the one Place to the other; *i. e.* the Angle wherein the *Rumb-line* must cut every *Meridian* in sailing from A to B, or from B to A; which Angle may be measured Mechanically.

II.

Two Lines, parallel to the *Meridian*, drawn through the given Places A and B, and continued till they meet the (graduated) *Equator*, or the top or bottom of the *Chart* (where the Degrees of *Longitude* are likewise marked) will intercept a Distance as AC, equal to the Difference of *Longitude* of the given Places.

III.

Two Lines, through A and B, parallel to the *Equator*, and continued till they meet the *East* or *West* Margin of the *Chart*, will there intercept a Line equal to CB; and the Degrees and Minutes marked on that *Margin*, will shew the Difference of *Latitude* of the Places A and B.

IV.

IV.

THE Line AB, applied to the *Equator* as to a Scale, will not immediately shew the *true* Distance on a Rumb-line between A and B, but, by means of it, that Distance may be found. For the Line CB is not equal to the Arc of a *Meridian* between the *Parallels* of AB, but only an *Artificial* Representative of it. In like manner AB is not the *natural*, but *artificial* Distance on the Rumb between A and B. But as AB and CB are increased above their *natural* Magnitude in the same Proportion, if a Line (L) be taken in the same Ratio to AB on the *Chart*, as the *natural* Length of the Arc of a Meridian, intercepted by the *Parallels* of A and B (to be found in *Tab. III.*) hath to the *artificial* Length CB; that Line (L) applied to the *Equator*, or to one of its *Parallels* on the *Chart*, will shew the *true* Distance required.

PRACTICALLY thus; In CB, take Cb equal to the *natural* Length of the Arc of the Meridian between the *Parallels* of A and B, and through b draw ba parallel to BA, meeting AC in a, so shall ab be the *natural* Distance in the Rumb-line. 6 El. 4.

FROM these *Remarks*, the Questions usually proposed in *Mercator's* Sailing, may be solved either upon a *Chart*, or by a very simple geometrical Construction. As also the Reasons of the following *Arithmetical Solutions* may be easily understood.

II.

Arithmetical Solutions of the Cases of MERCATOR'S Sailing, on the Spheroid of the Earth.

THE Quantities which can enter into a Question of *Mercator's* Sailing, are no more than four, viz. *The Difference of Latitude*; *the Difference of Longitude*; *the Angle of the Course*; and *the Distance sailed*. For in the Triangle ABC (*Fig. III.*) the Angle at

C

C is a Right-Angle, and CAB is the Complement of ABC, the Angle of the Course. Setting aside these two, there remain BC, AC, ABC, and AB. Any two of which being *given*, the other two may be *required*. Whence there arise, in all, six Cases of Mercator's Sailing; for the *Combinations* of two things in four, are six. But here it is to be observed,

I.

THAT when the *Difference of Latitude* is said to be *given*, it is understood that *both Latitudes* are known; and when the *Difference of Latitude* is *sought*, it is understood that *one Latitude* is given with the Position of the other, because otherwise it could not be known to what Part of the *Quadrant* the *Difference of Latitude* (whether *given* or *sought*) did belong; and therefore, in this Method of Solution, the Question would remain altogether *indeterminate*.

II.

ONE of the six Cases, just now mentioned, is not to be solved in this Method, *viz.* When from *ab*, AC (the *Distance run*, and *Difference of Longitude*) given, it is required to find *cb* and ABC, the *Difference of Latitude* and *Course*. In *Plain Sailing* the Solution would be easy, because *ab* is not there different from AB; but in *Mercator's Sailing*, seeing *cb* is unknown, its Ratio to CB will be unknown likewise; and therefore the Ratio of *ab* to AB, and the Line AB it self. Whence the Triangle ABC cannot be constructed nor resolved. The Cases therefore of *Mercator's Sailing* to be resolved, are the five following.

CASE

CASE I.

Given the Difference of Latitude, and Difference of Longitude, to find the Course and Distance run.

EXAMPLE.

$$\begin{array}{l} \text{Given} \left\{ \begin{array}{l} \text{N. Lat. of A. } 38^\circ \text{, Merid. Pts } 2422 \\ \text{N. Lat. of B. } 5^\circ \text{, Merid. Pts } 294 \\ \text{(Whence BC=diff. } \dots = 2128) \\ \text{AC (Diff. of Longit.)=43^\circ = 2580} \end{array} \right. \end{array}$$

Required,

1. The Course ABC.
2. Distance (AB) *ab*.

$$1. \left\{ \begin{array}{l} \text{BC : AC} \\ 2128 : 2580 \end{array} \right\} :: \left\{ \begin{array}{l} \text{R : Tang. ABC.} \\ 100000 : 121241 \end{array} \right\} = \text{Tang. } 50^\circ. 29' \text{ required.}$$

$$\begin{array}{l} \text{Or by Logarithms} \left\{ \begin{array}{l} 13.41162 = \text{Log. } 2580 + \text{Log. Rad.} \\ 3.32797 = \text{Log. } 2128 \\ \text{Diff. } 10.08365 = \text{Log. Tang. } 50^\circ. 29'. \end{array} \right. \end{array}$$

$$2. \left\{ \begin{array}{l} \text{R : Sec. } 50^\circ. 29'. \\ 100000 : 157158 \end{array} \right\} :: \left\{ \begin{array}{l} \text{BC : AB.} \\ 2128 : 3344 \end{array} \right\} = \text{the Artificial Dist.}$$

To reduce which to the *natural* Distance ; from the Number of Minutes in 38° , which (*Tab. III.*) is 2240, take the Number of Minutes in 5° . which is 293, there remain 1947. Say therefore,

$$\left\{ \begin{array}{l} \text{BC : } bc \\ 2128 : 1947 \end{array} \right\} :: \left\{ \begin{array}{l} \text{AB : } ab \\ 3344 : 3059 \end{array} \right\} = \text{the Distance required.}$$

CASE

CASE II.

Given the Difference of Latitude and Course; to find the Difference of Longitude and Distance run.

EXAMPLE.

$$\begin{array}{l} \text{Given } \left\{ \begin{array}{l} \text{S. Lat. A} \dots 25^\circ \dots \text{Merid. Pts} \dots 1518 \\ \text{N. Lat. B} \dots 30^\circ \dots \text{Merid. Pts} \dots 1850 \end{array} \right. \quad \begin{array}{l} \text{Required,} \\ 1. \text{ AC.} \\ 2. \text{ (AB) } ab. \end{array} \\ \quad \quad \quad \text{(Whence BC} \text{---Sum} \dots = 3368) \\ \quad \quad \quad \text{The Angle ABC} = 43^\circ. \end{array}$$

$$1. \left\{ \begin{array}{l} R : \text{Tang. ABC} \\ 100000 : 93251 \end{array} \right\} :: \left\{ \begin{array}{l} BC : AC \\ 3368 : 3141' = 52^\circ. 21' = \text{Diff. of} \\ \text{Longit. required.} \end{array} \right.$$

$$\begin{array}{l} \text{By Logarithms } \left\{ \begin{array}{l} 3.52737 = \text{Log. } 3368. \\ 9.96965 = \text{Log. Tang. } 43^\circ. \\ \hline 13.49702 = \text{Sum} - \text{Log. Rad.} = \text{Log. } 3141. \end{array} \right. \end{array}$$

2. To find (AB) *ab*, say, as in the former Example,

$$\left\{ \begin{array}{l} R : \text{Sec. } 43^\circ \\ 100000 : 136733 \end{array} \right\} :: \left\{ \begin{array}{l} BC : AB \\ 3368 : 4605 \text{ the Artificial Dist.} \end{array} \right.$$

THEN, to 1470 (the Minutes in 25° : on the Spheroid *Tab. II.*) adding 1765, the Minutes in 30° ; the Sum is 3235. Say therefore,

$$\left\{ \begin{array}{l} BC : bc \\ 3368 : 3235 \end{array} \right\} :: \left\{ \begin{array}{l} AB : ab \\ 4605 : 4423 = \text{the natural Distance required.} \end{array} \right.$$

If, in this Example, the Common Table of *Meridional Parts* had been used, the *Difference of Longitude* would have come out 3206', exceeding the truth by $1^\circ. 5'$; and the *Distance (ab)* would have been 4512, exceeding the *true Distance* by 89 Minutes, or Miles of the Equator.

C A S E III.

Given the Difference of Latitude and Distance run; to find the Difference of Longitude and Angle of the Course.

E X A M P L E.

Given, as in Examp. 2.	{	S. Lat. A. . 25°. . Merid. Pts. -	1518	Required; 1. AC. 2. Ang. ABC.	
		N. Lat. B. . 30°. . Merid. Pts.	1850		
		(Whence BC=Sum =	3368		
		And bc, from Tab. III. =	3235)		
		ab	- - - =	4423	

1. $\left\{ \begin{matrix} bc : BC \\ 3235 : 3368 \end{matrix} \right\} :: \left\{ \begin{matrix} ab : AB \\ 4423 : 4605 \end{matrix} \right\} = \text{the artificial Distance.}$

2. From the Square of AB, which is - 21206025
Take the Square of BC - - - - - 11343424

The Square Root of the Remainder - - 9862601, which is 3141, will be = AC the *Difference of the Longitude* sought.

OR by the *Logarithms*; Seing $ABq - BCq = \overline{AB+BC} \times \overline{AB-BC}$,
= ACq,

To the Log. of 7973 (=AB+BC) which is . . 3.90162
Add the Log. of 1237 (=AB-BC) which is . . 3.09237

6 99399

Half their Sum, viz. - - - - 3.49699 will be the Logarithm of 3141=AC, as before found.

3. $\left\{ \begin{matrix} AB : BC \\ 4605 : 3368 \end{matrix} \right\} :: \left\{ \begin{matrix} R : \text{Cof. ABC} \\ 100000 : \text{Cof. } 43^\circ \end{matrix} \right\} = \text{Angle of the Course.}$

F

C A S E

CASE IV.

Given the Difference of Longitude and Course; to find the Difference of Latitude and Distance sailed.

EXAMPLE.

$$\begin{array}{l} \text{Given } \left\{ \begin{array}{l} \text{N. Lat. of B} \dots 54^\circ \\ \text{Diff. of Long. AC. } 28^\circ = 1680' \\ \text{ABC.} \dots = 37^\circ \end{array} \right. \end{array} \quad \begin{array}{l} \text{Required,} \\ 1. \text{ (BC) } bc \\ 2. \text{ (AB) } ab. \end{array}$$

$$1. \left\{ \begin{array}{l} R : \text{Co-Tang. ABC} \\ 100000 : 132704 \end{array} \right\} :: \left\{ \begin{array}{l} AC : BC \\ 1680 : 2229 \end{array} \right.$$

Now the *Meridional Parts* for the given *Latitude* of B, viz. 54° , are 3803. Subtract therefore the Number last found, 2229, from 3803, and find in *Tab. I.* what Degree and Minutes of Latitude hath the *Meridional Parts* equal to the Remainder 1574; which (taking a proportional Part) is found to be $25^\circ. 50'.$ = the Latitude of A. So that the Difference of Latitude sought, is $28^\circ. 10'.$

2. To find AB, say,

$$\left\{ \begin{array}{l} S. ABC : R \\ 60181 : 100000 \end{array} \right\} :: \left\{ \begin{array}{l} AC : AB \\ 1680 : 2791 \end{array} \right.$$

3. In *Tab. III.* from the Minutes in 54° . viz. $\dots 3195$
Take the Minutes in $\dots 25^\circ. 50'.$ $\dots 1520$

The Remainder $\dots 1675$ is the quantity of the Arc *bc*. Say therefore,

$$\left\{ \begin{array}{l} BC : bc \\ 2229 : 1675 \end{array} \right\} :: \left\{ \begin{array}{l} AB : ab \\ 2791 : 2098 \end{array} \right. = \text{the Distance sought.}$$

N.B.

N. B. If, in this *Example*, the Latitude of B had been 20° , then I must have subtracted from $BC=2229$, the Meridional Parts of 20° . viz. 1199, and the Remainder 1030 would have shewn that A was on the other side of the *Equator*, in the *Lat.* $17^{\circ}. 16'$.

C A S E V.

Given the Distance run, and Angle of the Course; to find the Difference of Latitude, and Difference of Longitude.

E X A M P L E.

Given $\left\{ \begin{array}{l} \text{N. Lat. of B} . 45^{\circ} \\ \text{Angle ABC} . . 23^{\circ} \\ \text{Distance run (ba)} 3700' \end{array} \right.$

Required,
1. (BC) *bc.*
2. AC.

1. $\left\{ \begin{array}{l} R : \text{Cof. ABC} \\ 100000 : 92050 \end{array} \right\} :: \left\{ \begin{array}{l} ba : bc \\ 3700 : 3406 \end{array} \right. = \text{the Arc of the Meridian intercepted by the Parallels through A and B.}$

2. In *Tab. III.* the Minutes for the given Latitude of B (45° .) are 2656. Therefore from $bc=3406$, I take 2656; and the remainder 750, applied to the same *Tab. III.* shews that A falls in $12^{\circ}. 46'$. *South Latitude*. I add then the *Meridional Parts* of $12^{\circ}. 46'$. (viz. 756.) to the *Meridional Parts* of 45° . (2976.) And the Sum 3732, is the Line BC on the *Chart*.

3. $\left\{ \begin{array}{l} bc : BC \\ 3406 : 3732 \end{array} \right\} :: \left\{ \begin{array}{l} ab : AB \\ 3700 : 4051 \end{array} \right. = AB.$

$$\begin{array}{rcl}
 4. \text{ As in Case III. to Log. } 7783 (=AB+BC) & \dots & 3.89115 \\
 \text{Add Log. } & \dots & 319 (=AB-BC) \dots 2.50379
 \end{array}$$

$$6.39494$$

$$\begin{array}{rcl}
 \frac{1}{2} \text{ the Sum } & \dots & 3.19747 \text{ is} \\
 \text{the Log. of } 1576=AC \text{ sought.} & &
 \end{array}$$

THE like Operations will serve for the same Cases in common *Mercator's Sailing*; only the Reduction of the Arcs *bc*, which is here performed by the help of *Tab. III.* is there done at once, by multiplying the Degrees in *bc* by 60; adding, if there are any, the odd Minutes.

I say nothing of *Parallel-Sailing*, having sufficiently explained it above, *Pag. 13.* nor of *Traverse Courses*, which being no more than *Combinations* of the *simple Cases*, are solved by the same Rules, though with more Labour.

I beg leave only, to subjoin the following Remarks upon the Method it self.

I.

As I insinuated (*Pag. 14.*) the Necessity of a Correction in our common *Charts* and *Tables*, for this general Reason, that, "*In the mixt Sciences, unavoidable Errors ought, by the Exactness of the Theoretical Part, to be kept within the narrowest Bounds possible;*" So we see from *Examp. H.* that the Errors of *Mercator's Chart* may sometimes considerably augment whatever others a Ship's *Reckoning* is subject to. And Cases may be imagined, in which the Errors would be still greater than in that Example.

II.

IF, from the Situation of two Places (*A* and *B*) with respect to *Longitude* and *Latitude*, the Angle of the *Course* is determined, as in
Case

C, D, E:) and he will see how high the *Error* of the *last Course*, as it involves all the preceeding *Errors* of the *Longitudes*, may sometimes rise.

III.

IN *Latitudes* under 28° , the *Plain Chart* is preferable to *Mercator's*. From 0° *Lat.* to 20° . the *Meridional Distances* of *Mercator* exceed those of the *Plain Chart*; which last exceed the *Truth*. From 20° . to 28° . the *Defect* of the *Meridional Distances* on the *Plain Chart* is less than the *Excess* of *Mercator's*. After this the said *Defect* increases quickly, so that, between 33° . and 34° . it is *double*, and at 38° . a little more than *triple* the *Excess*. Whence I infer, that for about a third part of the *Quadrant*, there is at least as much *Reason* for correcting the *Sea-Charts* now, as formerly.

F I N I S
10 AP 58



No. 100

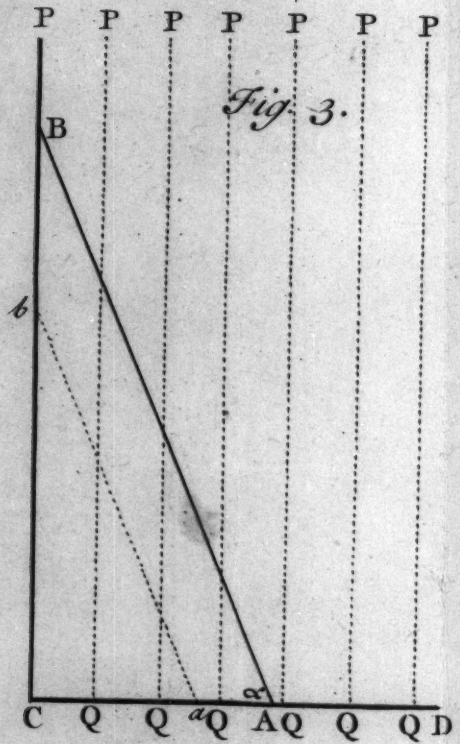
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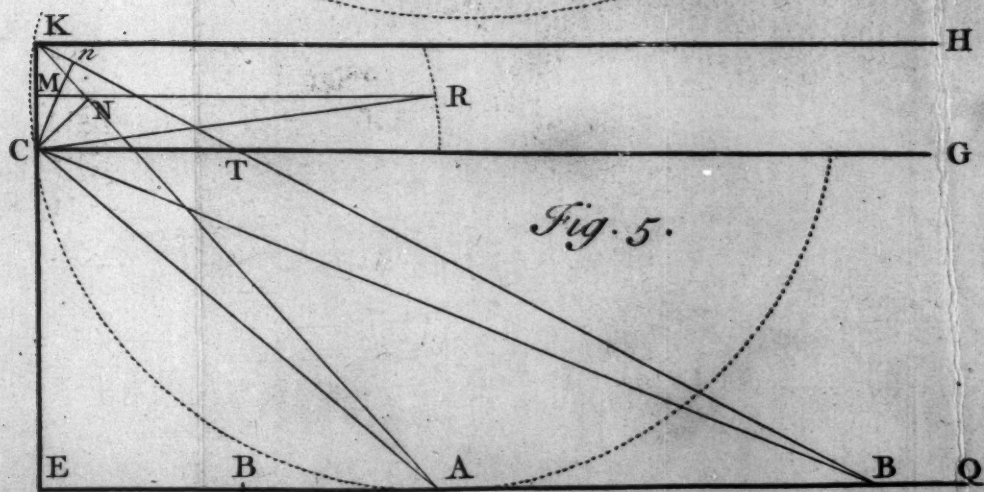
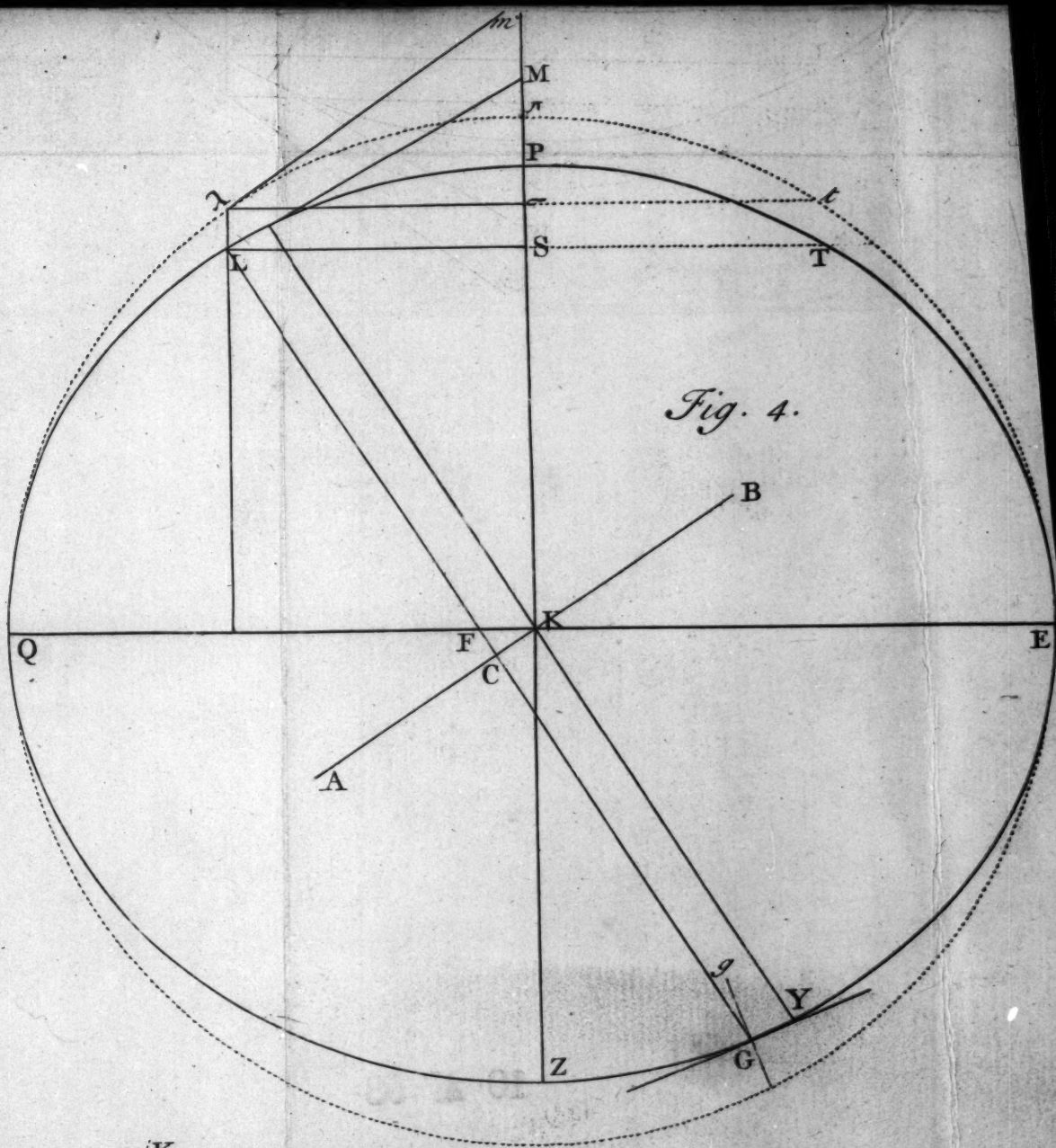


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